

• Logistic Equation (非線形)

$$\frac{dN}{dt} = (\varepsilon - \lambda N)N$$

λ : 混雑定数
 N に対して1次でない

初期条件 $(N, t) = (N_0, t_0)$ として解く

$$G(N) = \int_{N_0}^N \frac{dx}{x(\varepsilon - \lambda x)} = \int_{t_0}^t dt$$

t について

$$G(N) = t - t_0$$

N について

$$\begin{aligned} G(N) &= \int_{N_0}^N \frac{1}{\varepsilon} \left(\frac{1}{x} + \frac{\lambda}{\varepsilon - \lambda x} \right) dx \\ &= \frac{1}{\varepsilon} \left[\ln x - \ln(\varepsilon - \lambda x) \right]_{N_0}^N \\ &= \frac{1}{\varepsilon} \left\{ \ln \frac{N}{N_0} - \ln \left(\frac{\varepsilon - \lambda N}{\varepsilon - \lambda N_0} \right) \right\} \\ &= \frac{1}{\varepsilon} \ln \left(\frac{N}{\varepsilon - \lambda N} \cdot \frac{\varepsilon - \lambda N_0}{N_0} \right) \end{aligned}$$

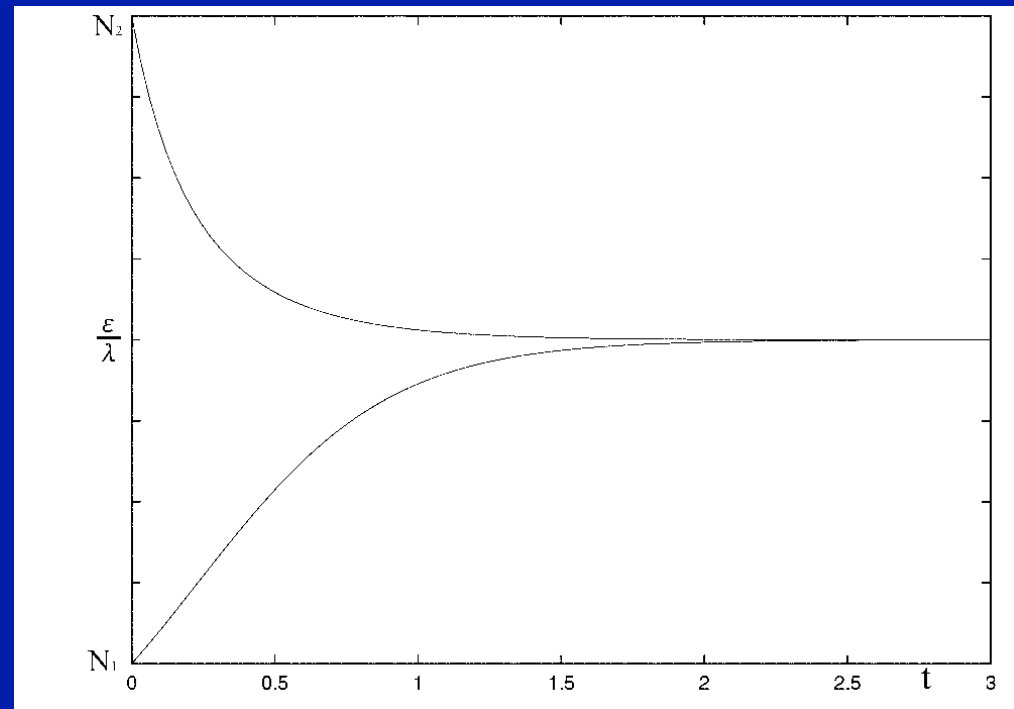
$$C = \frac{N_0}{\varepsilon - \lambda N_0} \quad \text{とおくと}$$

$$\frac{N}{\varepsilon - \lambda N} = C e^{\varepsilon(t-t_0)}$$

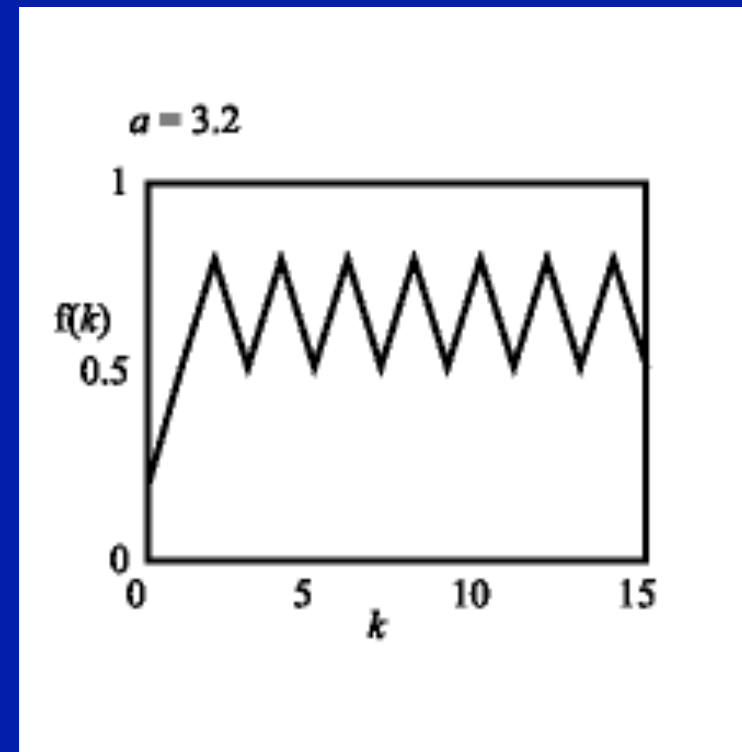
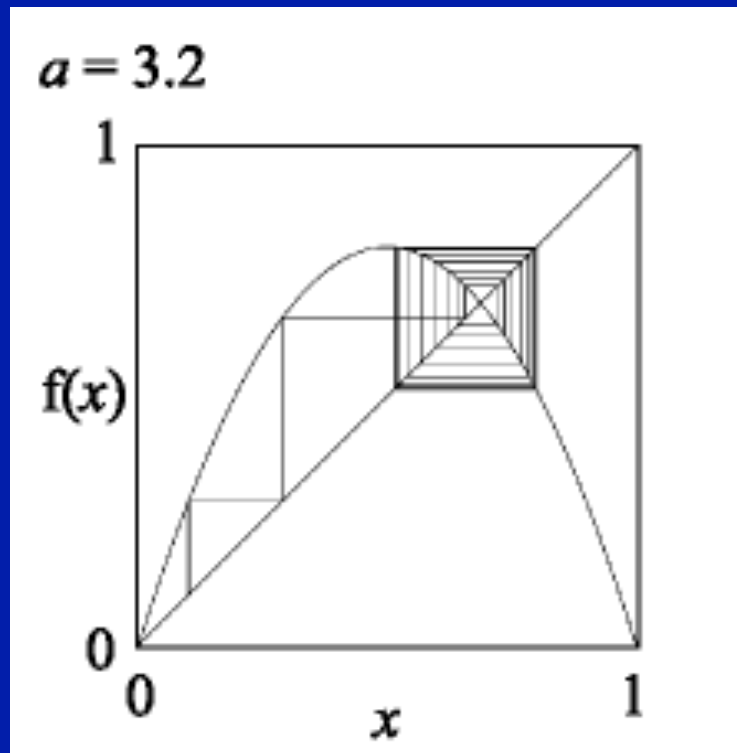
$$\begin{aligned} N(t) &= \frac{\varepsilon C e^{\varepsilon(t-t_0)}}{1 + \lambda C e^{\varepsilon(t-t_0)}} \\ &= \frac{\varepsilon N_0 e^{\varepsilon(t-t_0)}}{\varepsilon + \lambda N_0 \{e^{\varepsilon(t-t_0)} - 1\}} \end{aligned}$$

$t \rightarrow +\infty$ のとき

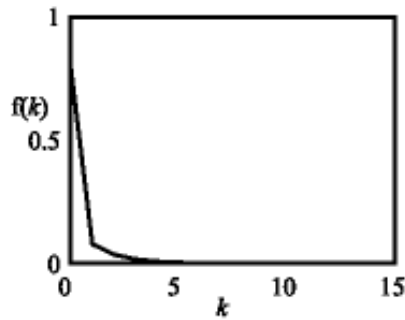
$$N(\infty) = \frac{\varepsilon}{\lambda}$$



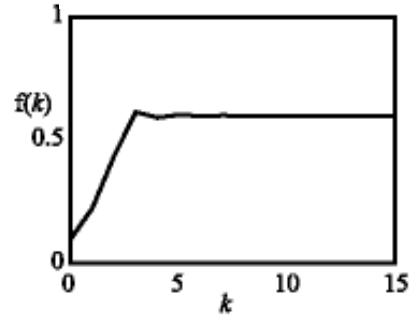
$$x(n + 1) = ax(n) \{1 - x(n)\}$$



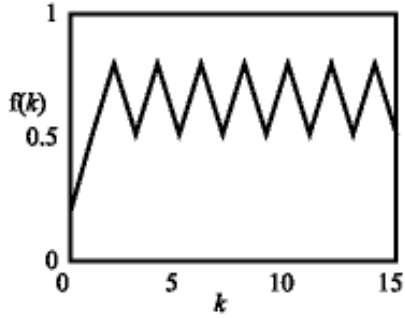
$a = 0.5$



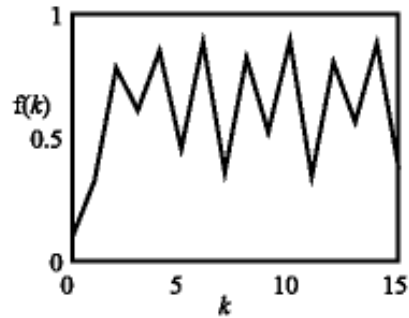
$a = 2.5$



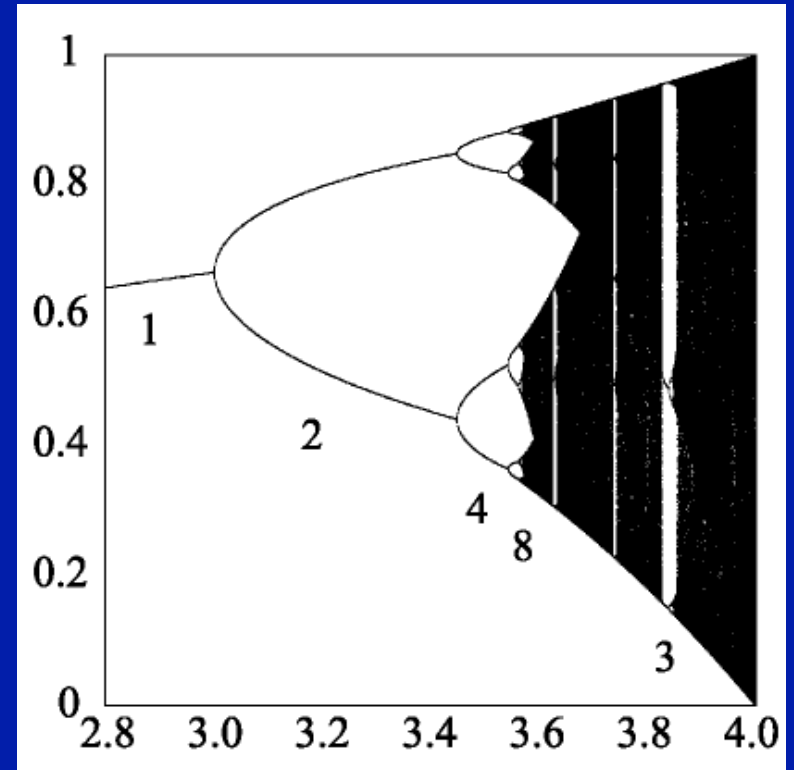
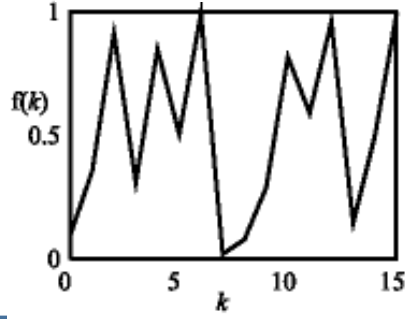
$a = 3.2$



$a = 3.58$



$a = 3.98$



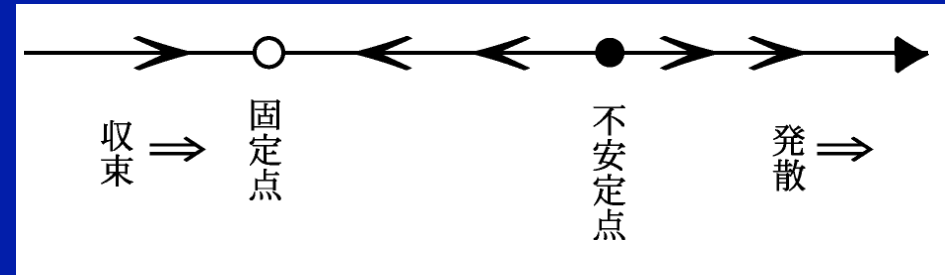
分岐図 ($x(0) = 0.1$)

(注) 数字は周期点を示す

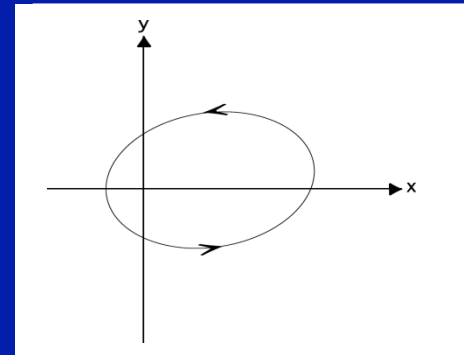
a の変化による解のふるまい

変数の数(次元) と微分方程式の解の挙動

一次元

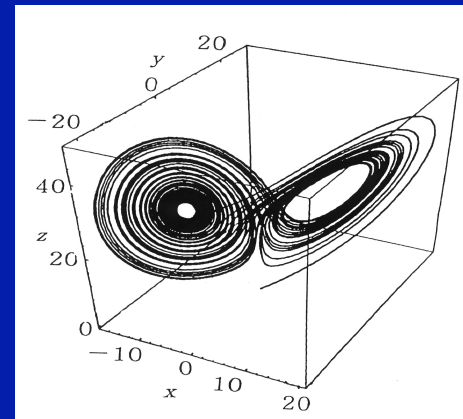


二次元



解曲線は交差
できない

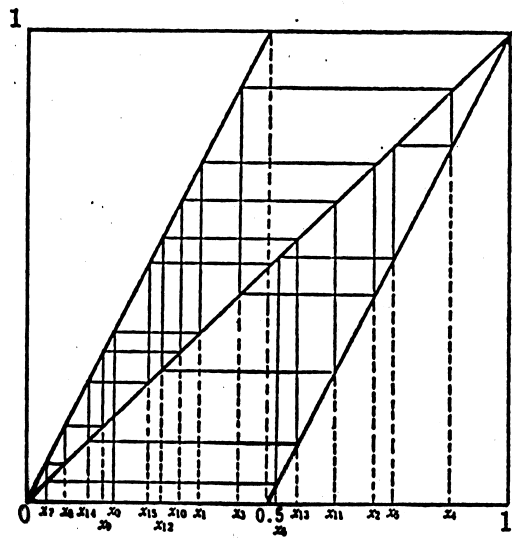
三次元



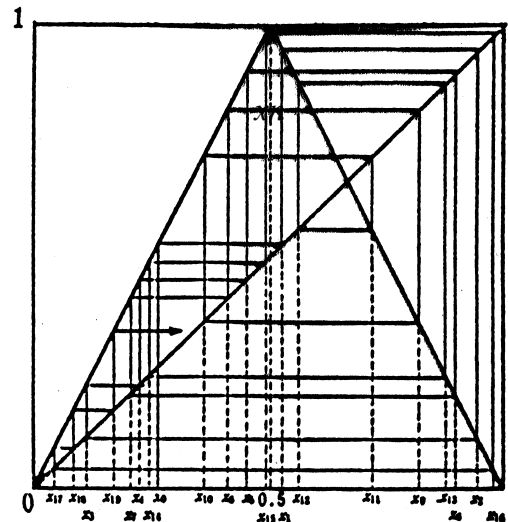
ストレンジ
アトラクター

決定論的方程式⇒サイコロ振り(非決定論的)

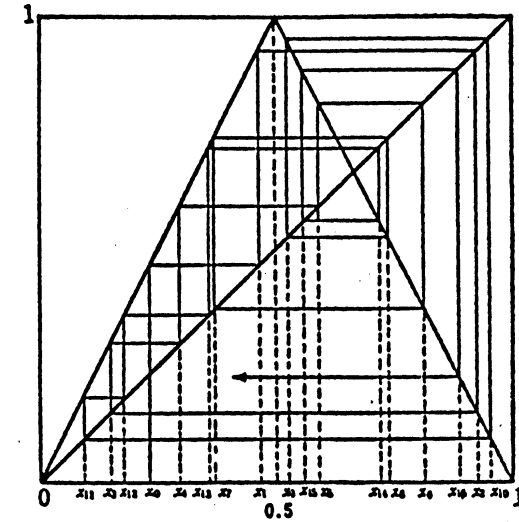
$$x_{n+1} = f(x_n)$$



$x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, \dots$
 $A A B A B B B A A A B A B A A \dots$



$x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, \dots$
 $A B B A A A B A A B A B B B A A B A A \dots$



$x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, \dots$
 $A A B A A B B A B B B A A A B B B \dots$