• Logistic Equation（非線形）

$$\frac{dN}{dt} = (\varepsilon - \lambda N)N$$

λ：混雑定数
Nに対して1次でない

初期条件(N,t)=(N_0,t_0)として解く

$$G(N) = \int_{N_0}^{N} \frac{dx}{x(\varepsilon - \lambda x)} = \int_{t_0}^{t} dt$$

tについて

$$G(N) = t - t_0$$

Nについて

$$G(N) = \int_{N_0}^{N} \frac{1}{\varepsilon}\left(\frac{1}{x} + \frac{\lambda}{\varepsilon - \lambda x}\right)dx$$

$$= \frac{1}{\varepsilon}\left[\ln x - \ln(\varepsilon - \lambda x)\right]_{N_0}^{N}$$

$$= \frac{1}{\varepsilon}\left\{\ln\frac{N}{N_0} - \ln\left(\frac{\varepsilon - \lambda N}{\varepsilon - \lambda N_0}\right)\right\}$$

$$= \frac{1}{\varepsilon}\ln\left(\frac{N}{\varepsilon - \lambda N} \bullet \frac{\varepsilon - \lambda N_0}{N_0}\right)$$

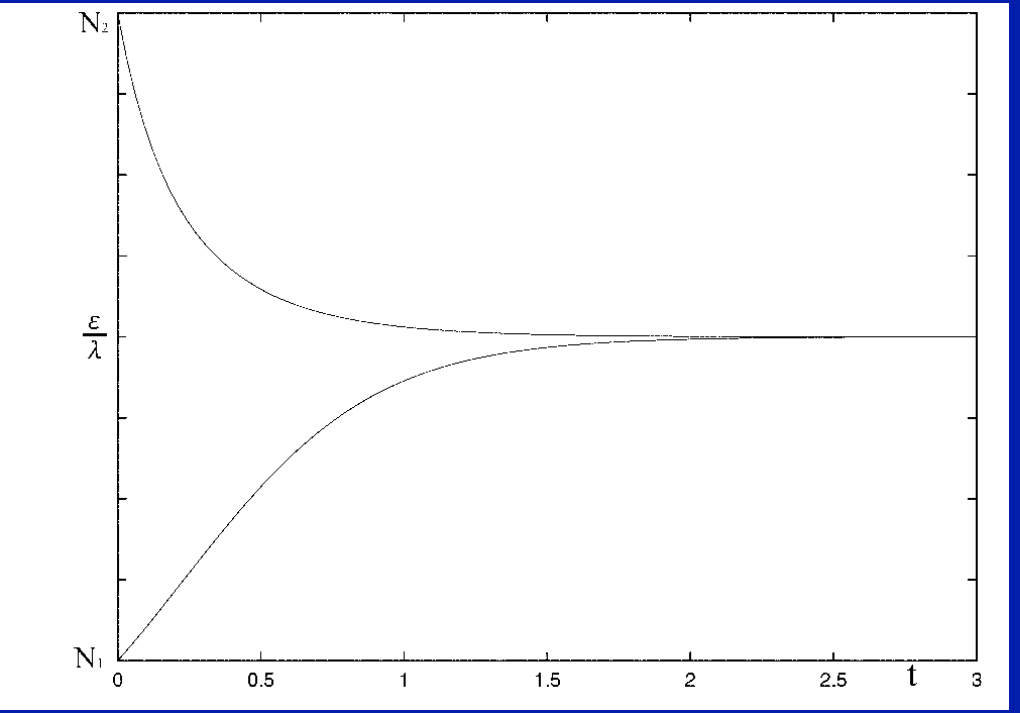
$$C = \frac{N_0}{\varepsilon - \lambda N_0}$$

とおくと

$$\frac{N}{\varepsilon - \lambda N} = Ce^{\varepsilon(t-t_0)}$$

$$N(t) = \frac{\varepsilon C e^{\varepsilon(t-t_0)}}{1 + \lambda C e^{\varepsilon(t-t_0)}}$$

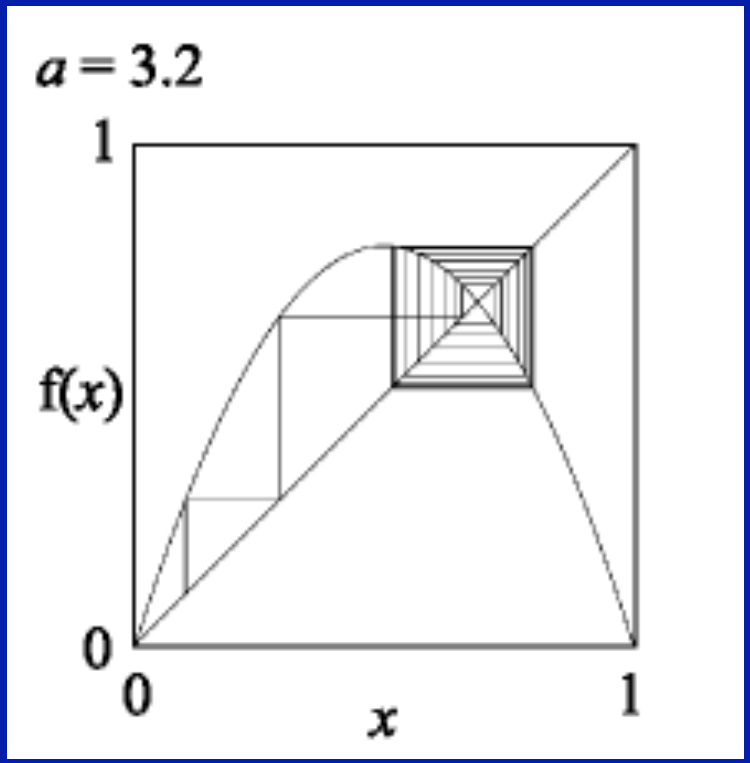
$$= \frac{\varepsilon N_0 e^{\varepsilon(t-t_0)}}{\varepsilon + \lambda N_0 \{e^{\varepsilon(t-t_0)} - 1\}}$$

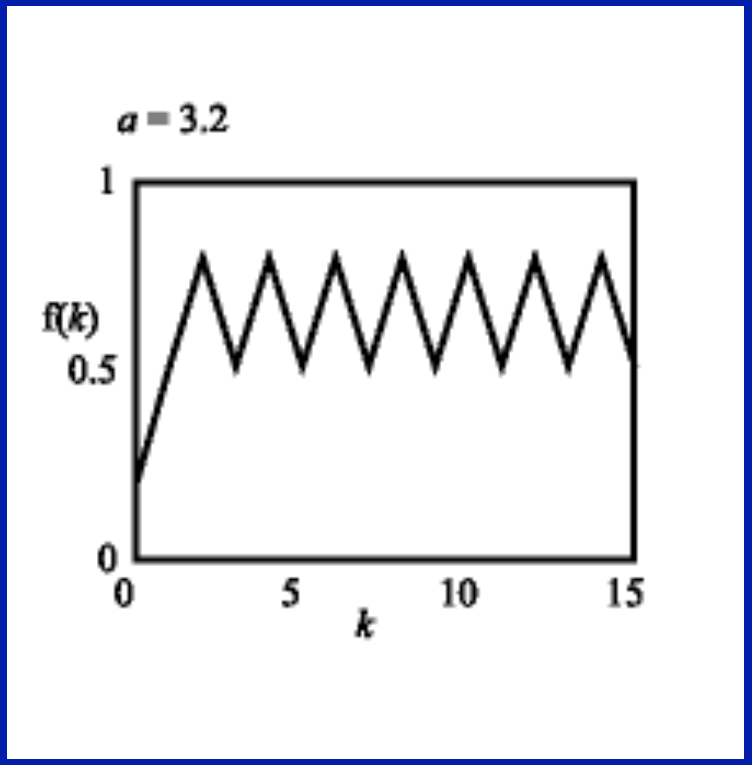


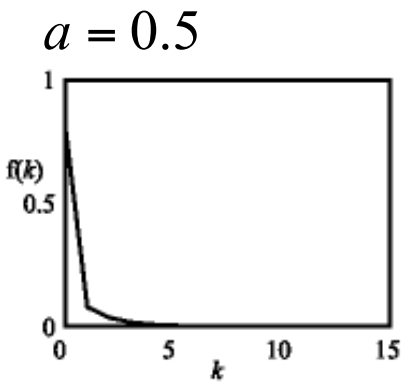
$t \to +\infty$のとき

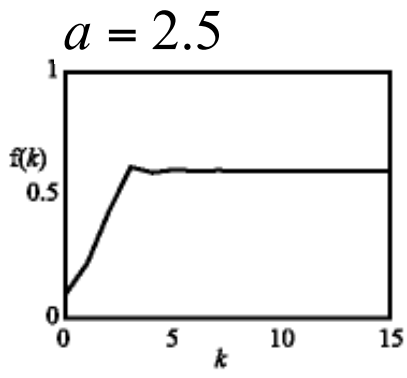
$$N(\infty) = \frac{\varepsilon}{\lambda}$$

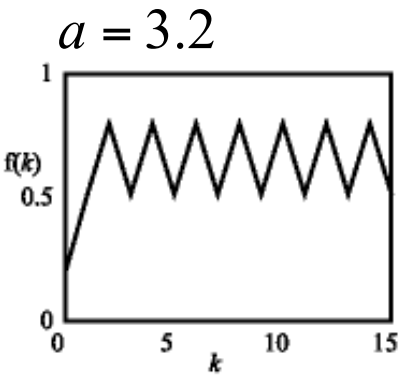
$$x(n+1) = ax(n)\{1 - x(n)\}$$

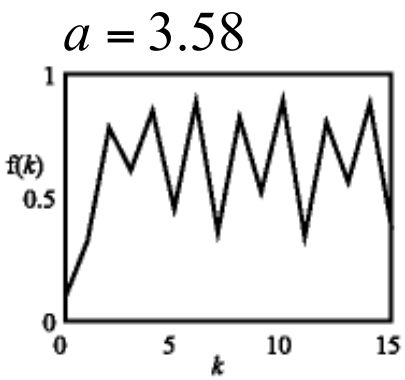


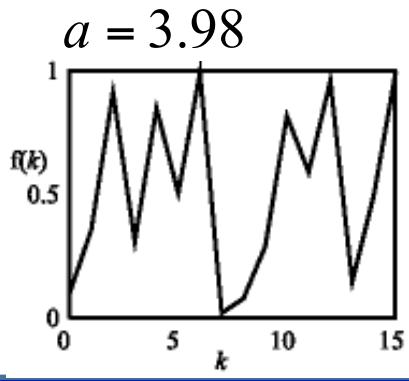




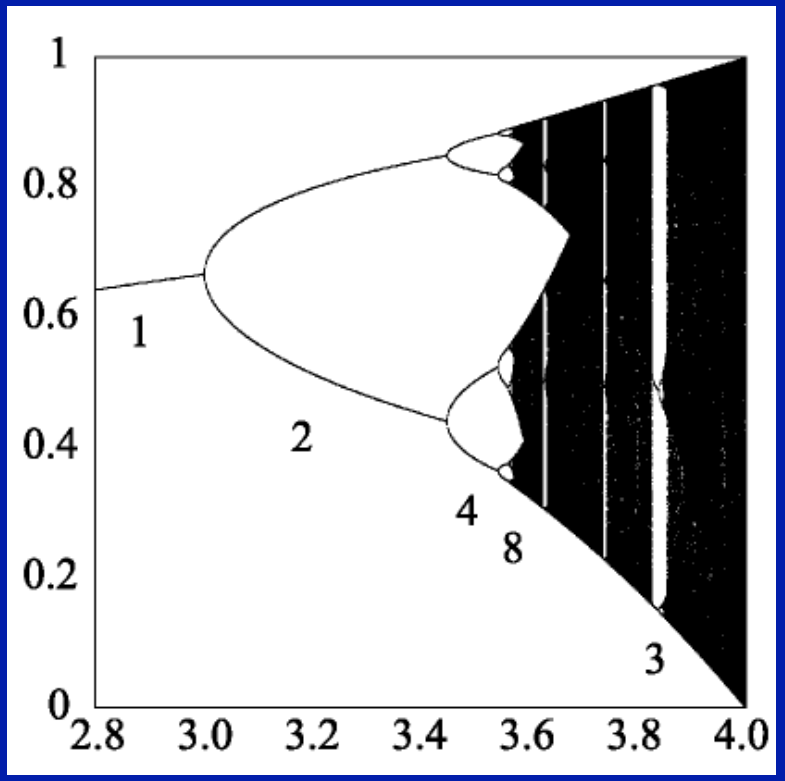








a の変化による解のふるまい

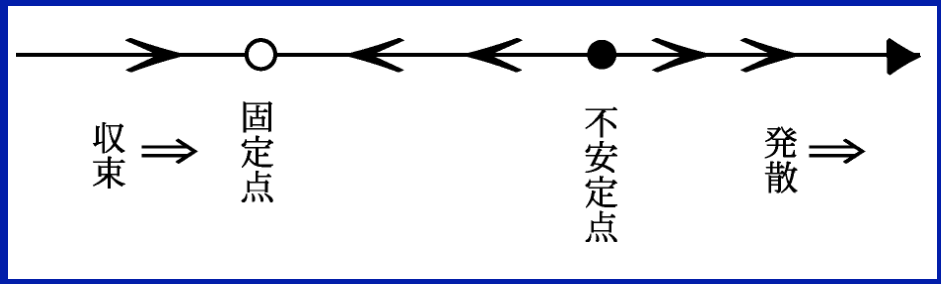


分岐図 ($x(0) = 0.1$)

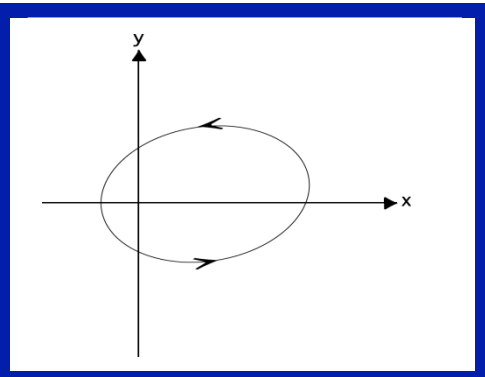
(注)数字は周期点を示す

変数の数(次元)と微分方程式の解の挙動

一次元

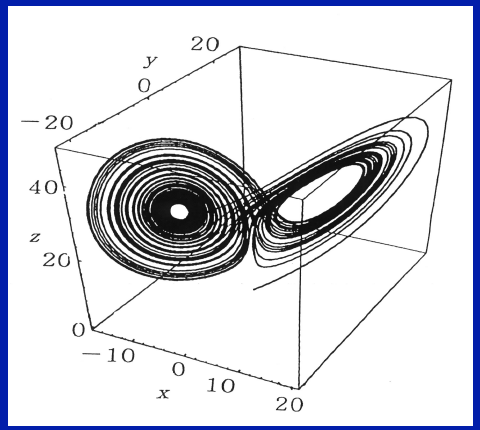


二次元



解曲線は交差できない

三次元



ストレンジアトラクター

決定論的方程式⇒サイコロ振り（非決定論的）

$$x_{n+1} = f(x_n)$$