計算科学が拓く世界
地球・惑星・宇宙と計算科学２
宇宙プラズマ粒子シミュレーション

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計算科学が拓く世界　第13回　2016年7月13日 第5限

ブラゾフ方程式 → MHD方程式

速度分布関数 $f(\mathbf{x},\mathbf{v},t)$

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}}+\frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}}=\left.\frac{\partial f}{\partial t}\right|_{\mathrm{coll}}$$

$$n_\alpha = \int F_\alpha d\underline{v}$$

$$\underline{u}_\alpha = \frac{1}{n_\alpha}\int \underline{v}F_\alpha d\underline{v} = \langle \underline{v} \rangle$$

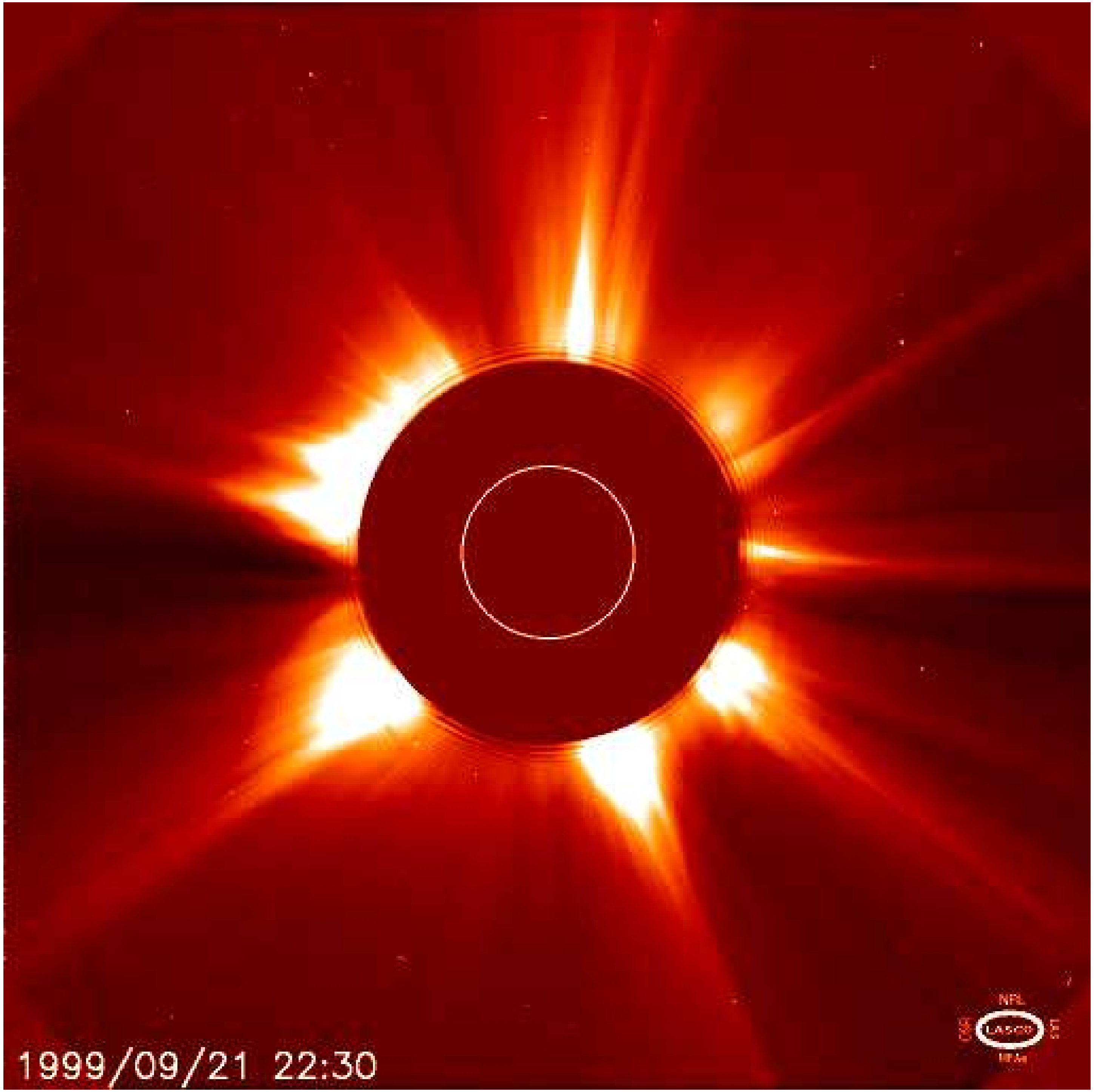
speed of light $\rightarrow \infty$

electron inertia $\rightarrow 0$

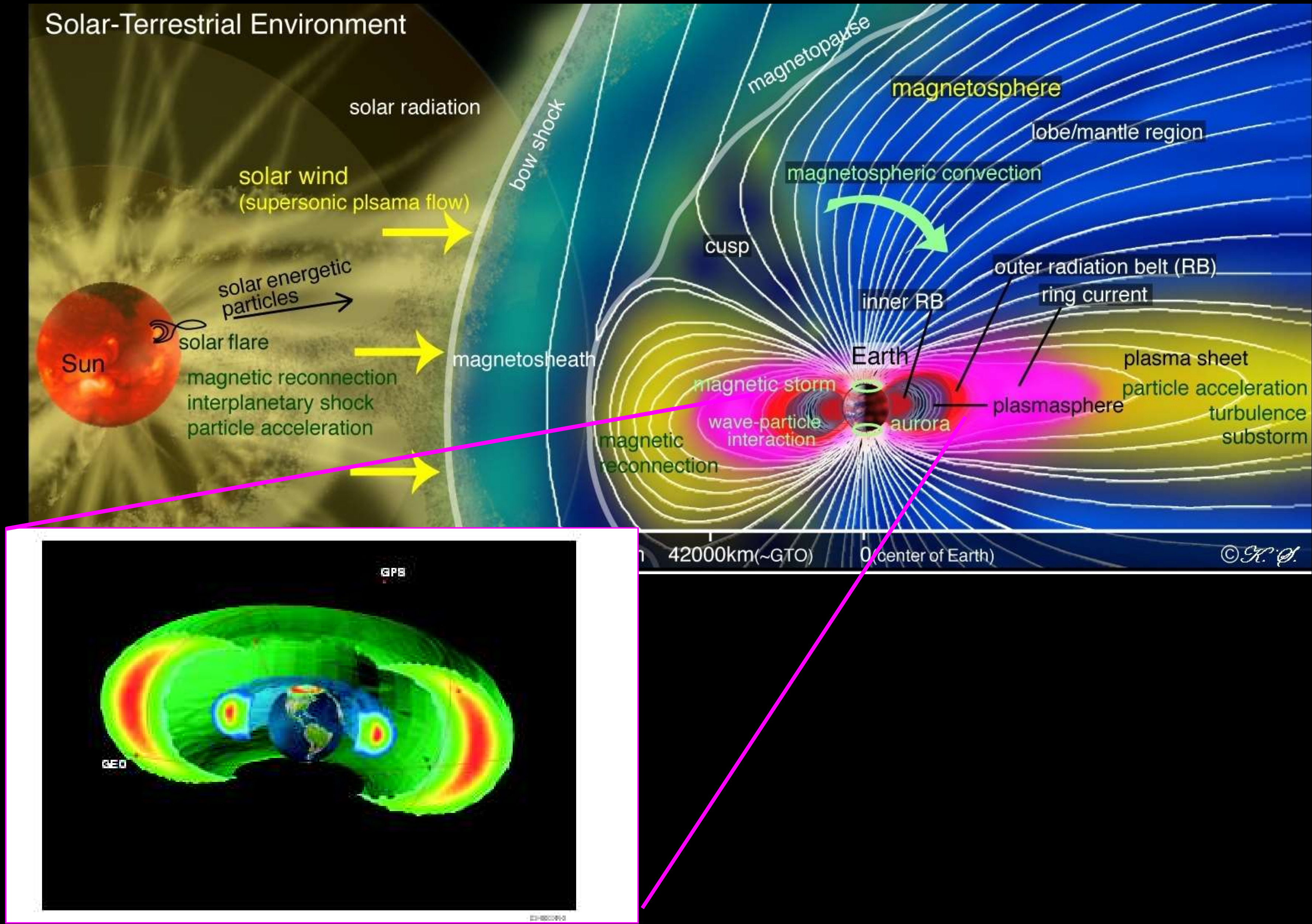
$$\overleftrightarrow{P}_\alpha = n_\alpha m_\alpha \langle \overleftrightarrow{\underline{v}}\,\overleftrightarrow{\underline{v}} \rangle$$

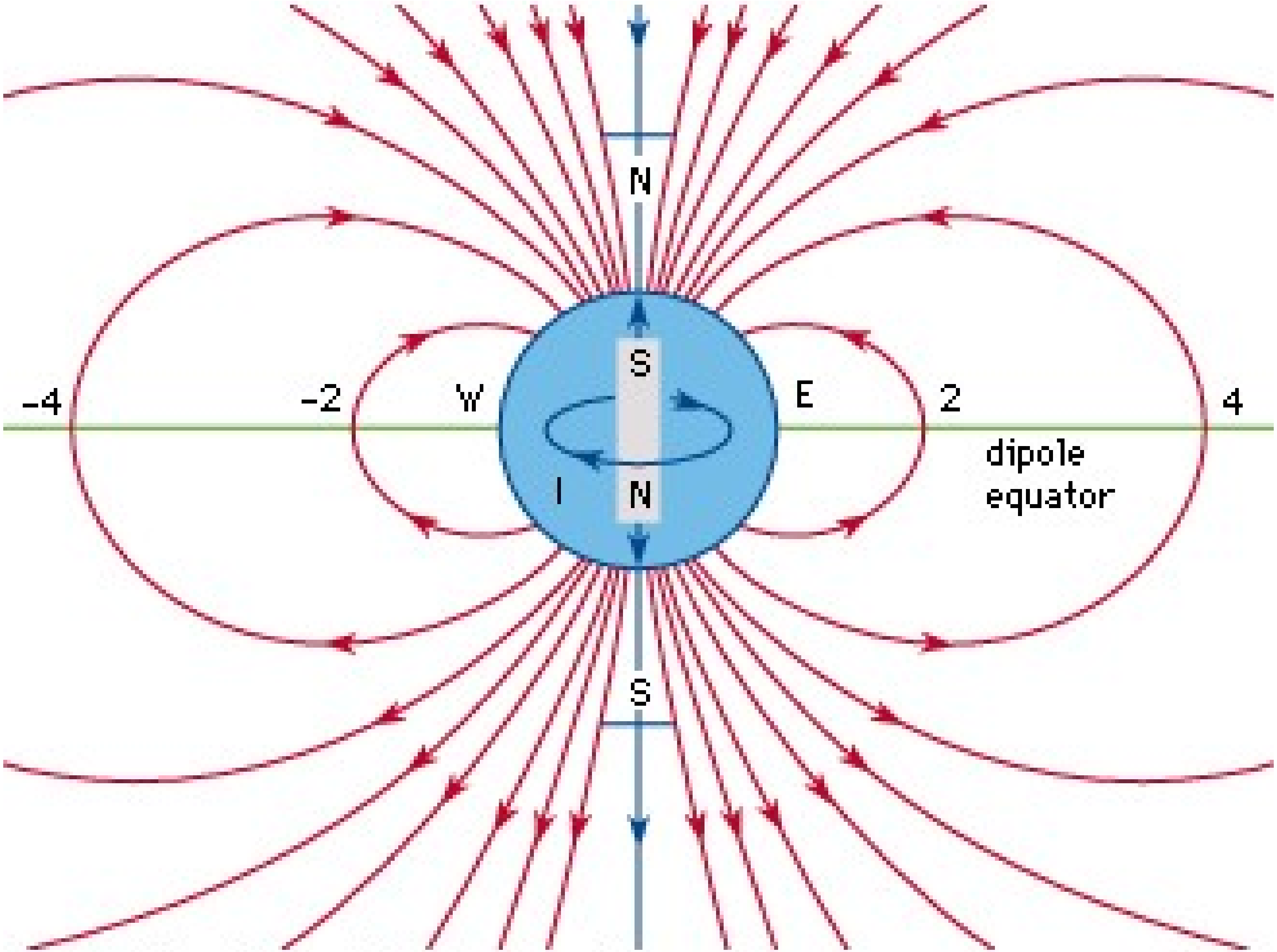
$$m_\alpha n_\alpha \frac{d\underline{u}_\alpha}{dt} = q_\alpha n_\alpha \left(\underline{E} + \underline{u}_\alpha \times \underline{B}\right) - \nabla \cdot \underline{\underline{P}}_\alpha + \underline{R}_\alpha$$

太陽風

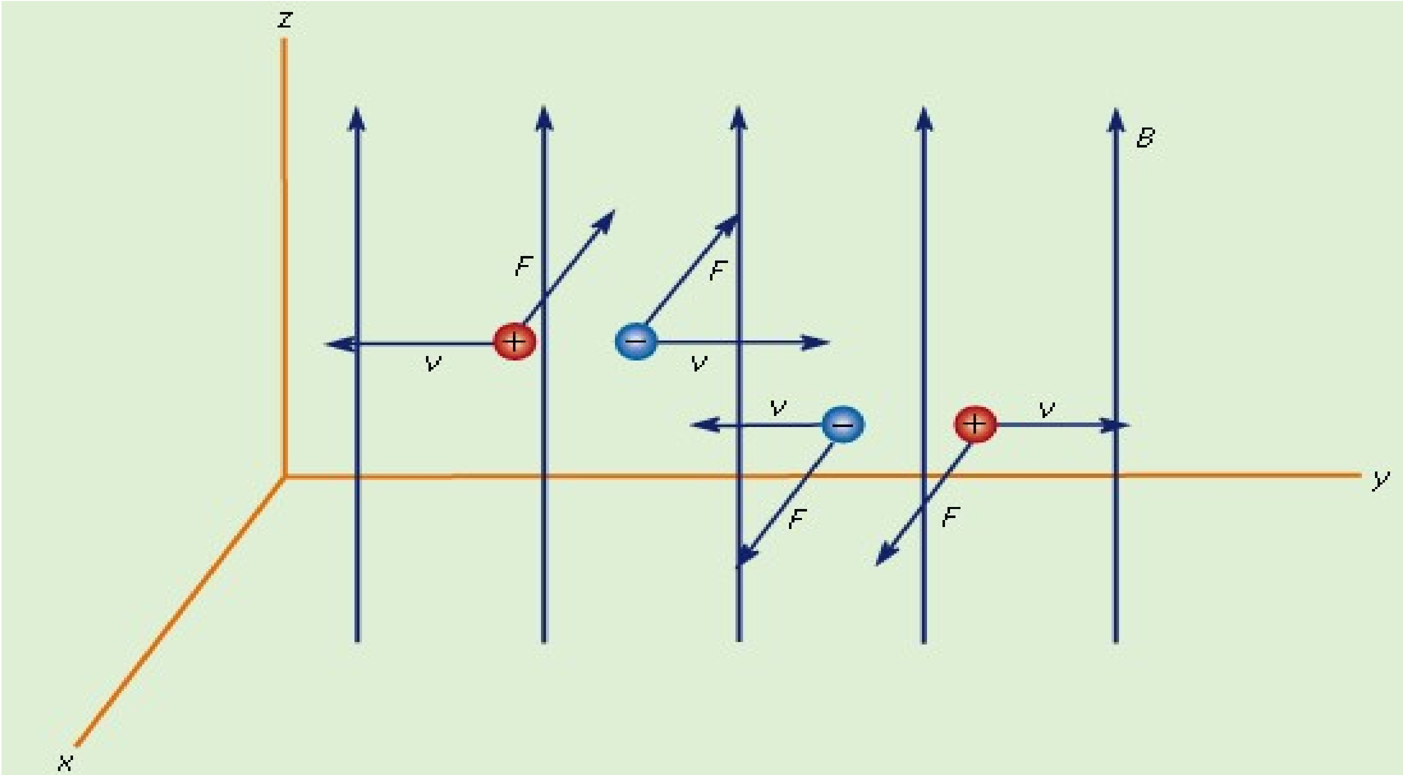


Geospace


Solar-Terrestrial Environment
solar radiation
solar wind
(supersonic plsama flow)
solar energetic
particles
solar flare
Sun
magnetic reconnection
interplanetary shock
particle acceleration
bow shock
magnetosheath
magnetopause
magnetosphere
lobe/mantle region
magnetospheric convection
cusp
outer radiation belt (RB)
ring current
inner RB
Earth
plasma sheet
particle acceleration
turbulence
substorm
magnetic storm
plasmasphere
wave-particle
interaction
aurora
magnetic
reconnection
42000km(~GTO)
0 (center of Earth)
GPS
GEO


N
S
W
E
-4
-2
2
4
I
N
dipole
equator
S

Motion of Charged Particles （Lorentz Force）

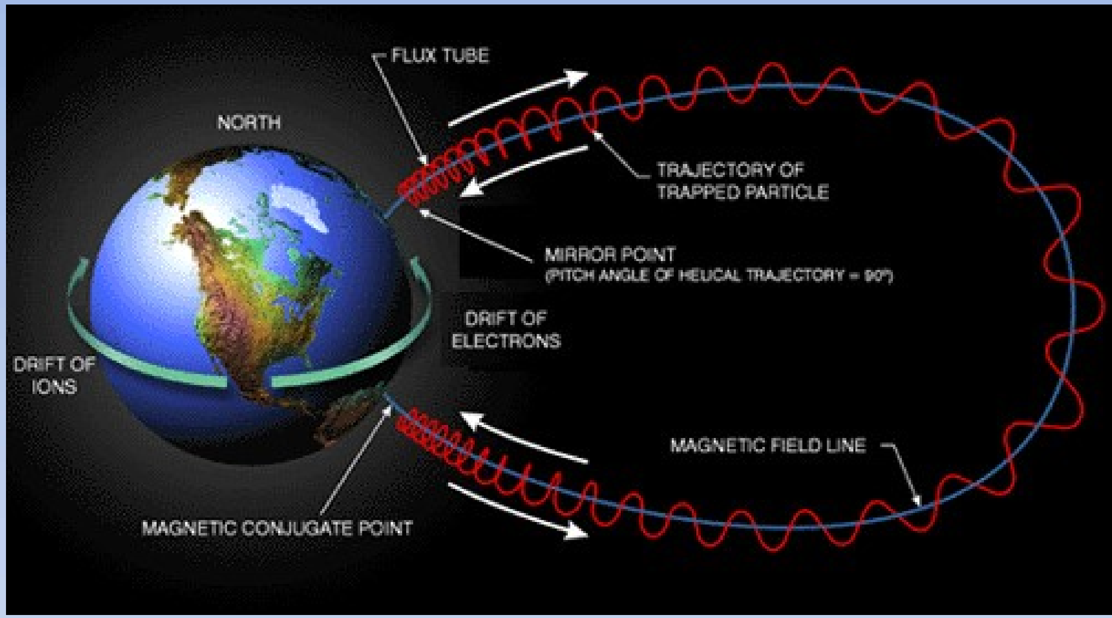


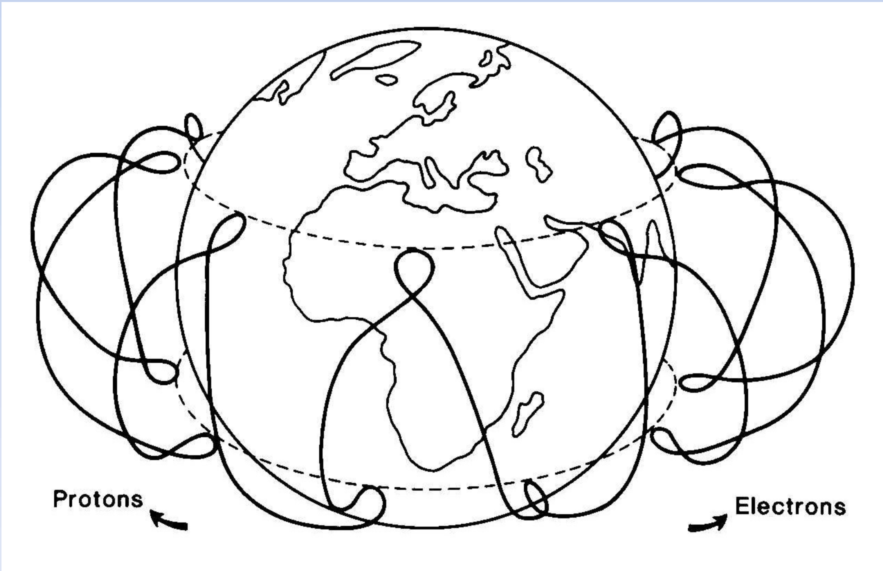
Equation of Motion of Relativistic Electrons

$$m_0 \frac{\mathrm{d}(\gamma \boldsymbol{v})}{\mathrm{d}t} = -e[\boldsymbol{E}_w + \boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{B}_w)]$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Motion of Charged Particles in Dipole Magnetic Field



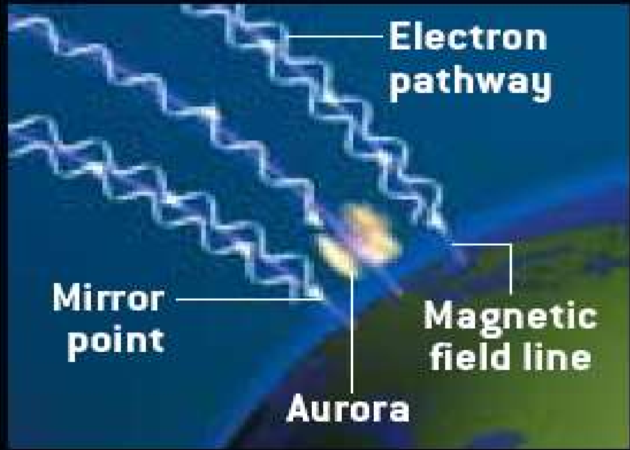


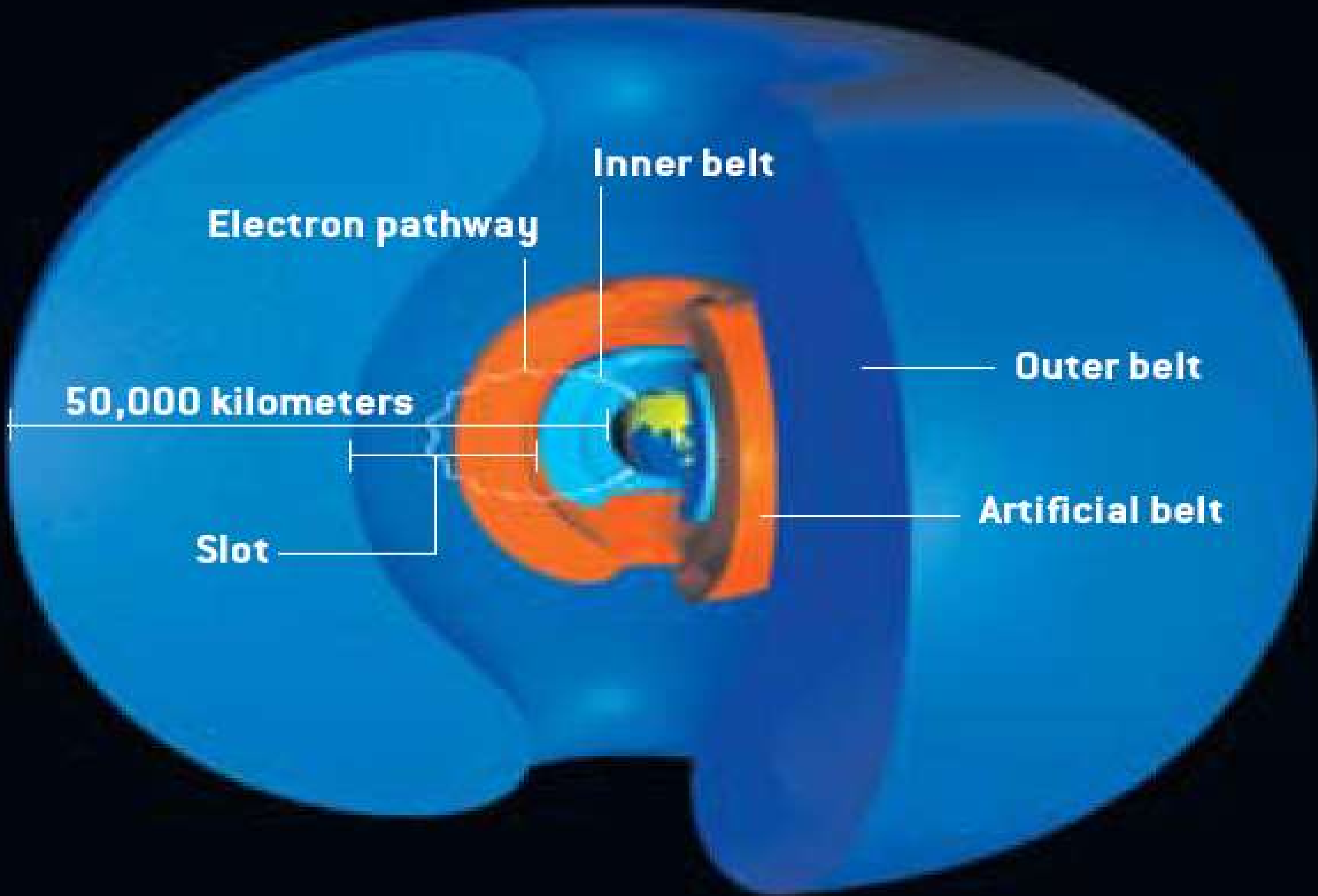
VAN ALLEN RADIATION BELTS

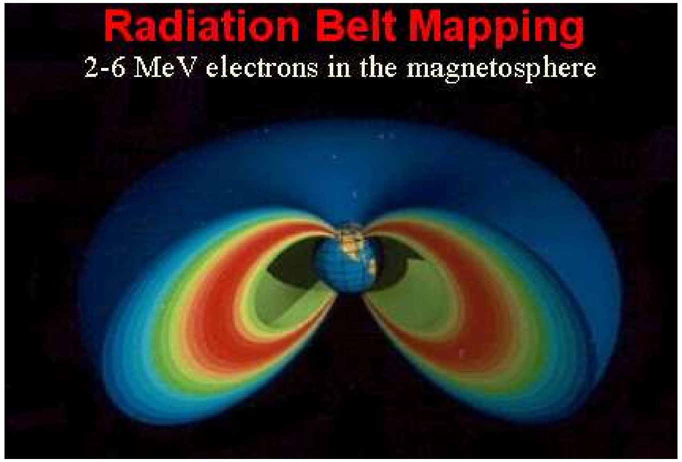
Inner electron belt:
600 to 5,000 kilometers

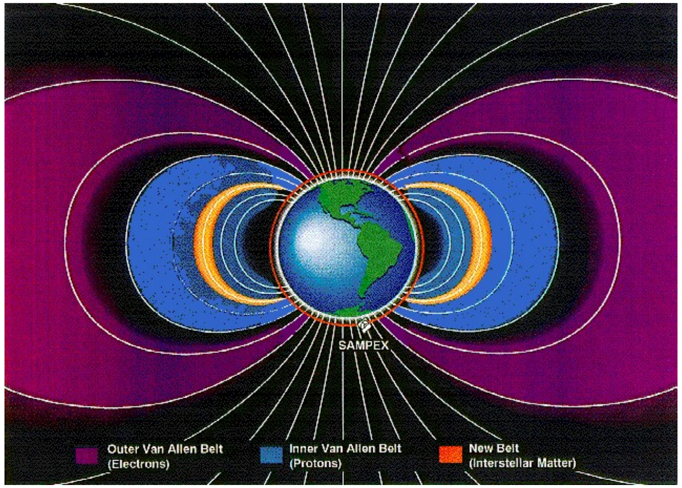
Slot (safety zone):
6,000 to 12,000 kilometers

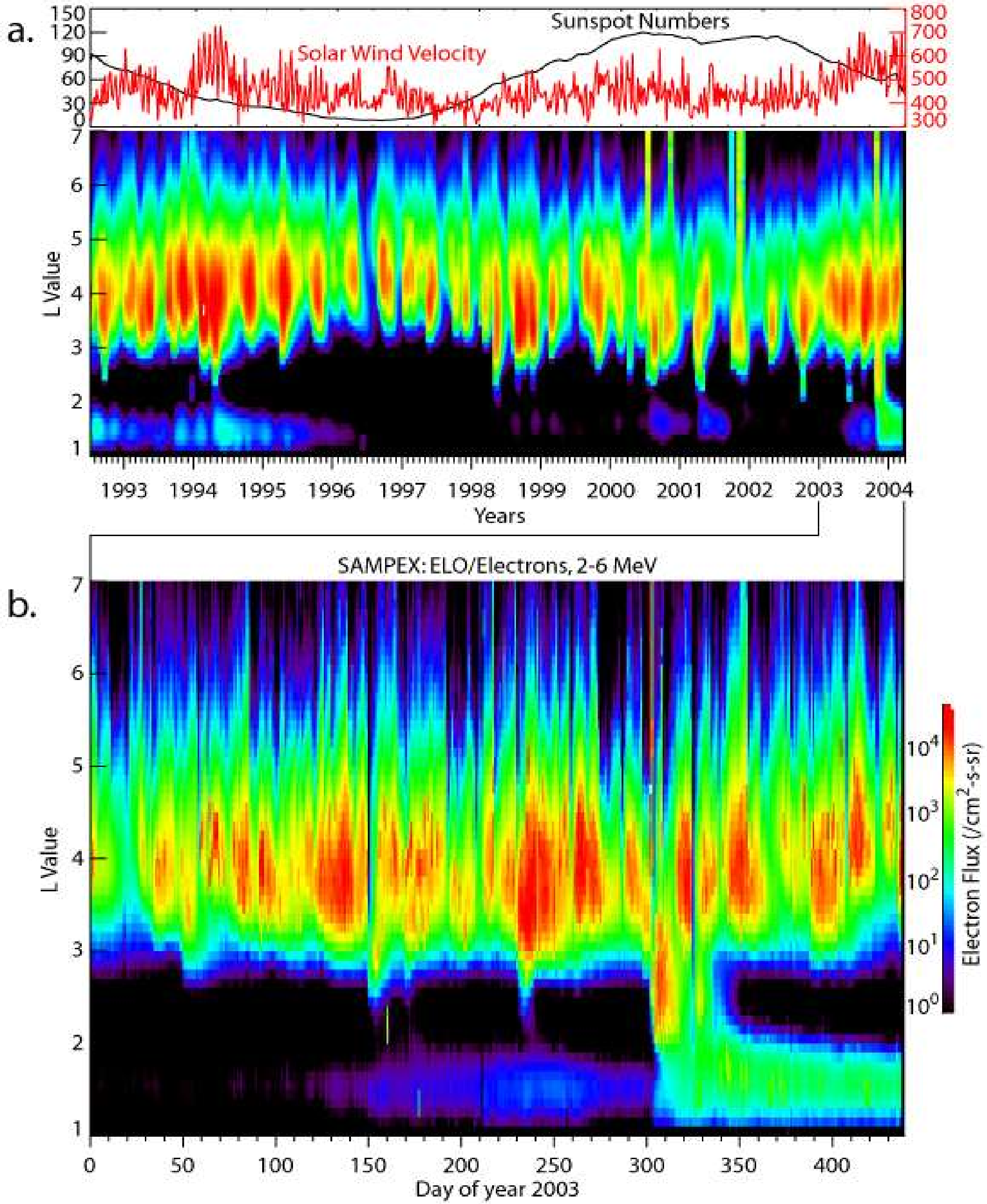
Outer electron belt:
20,000 to 50,000 kilometers





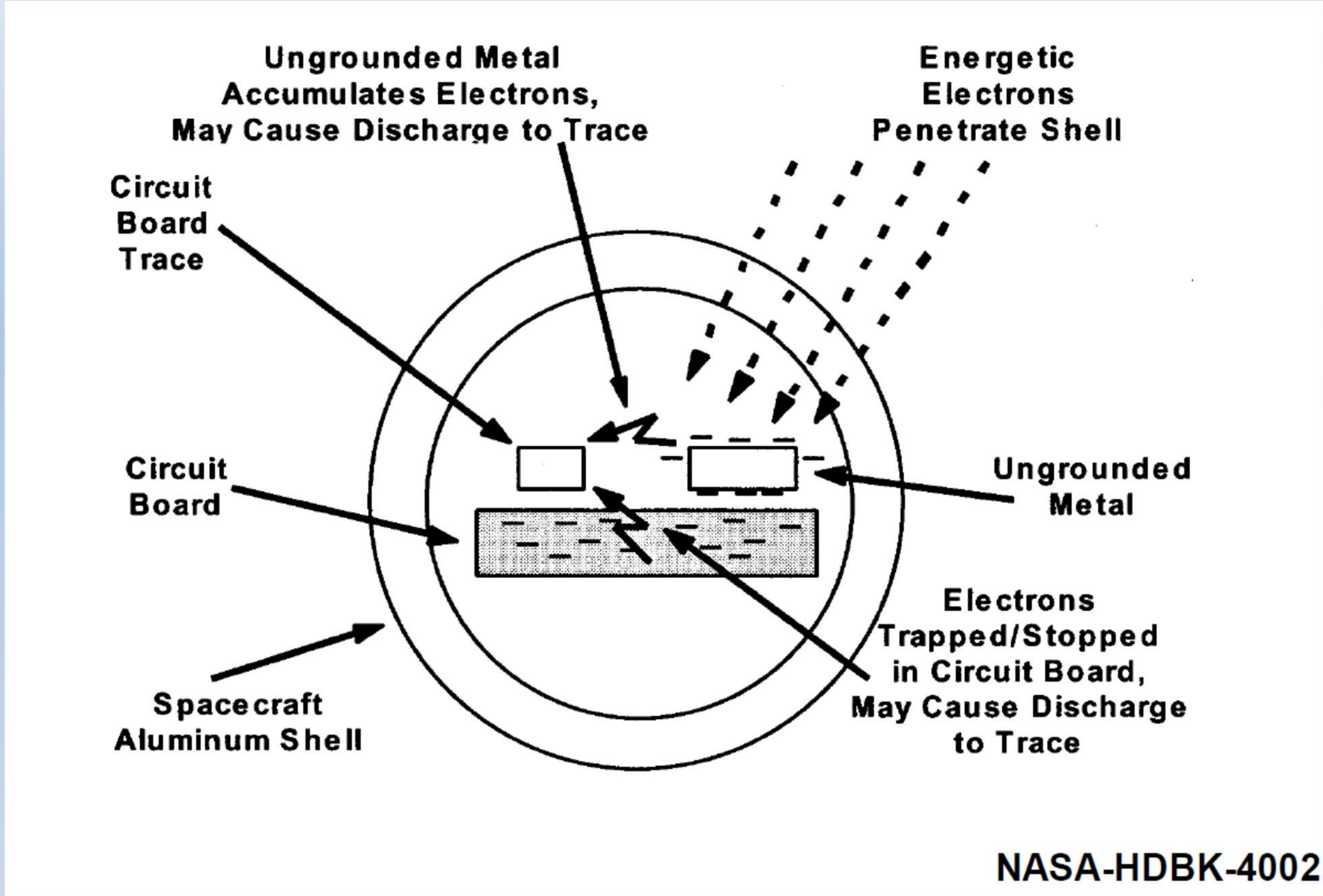




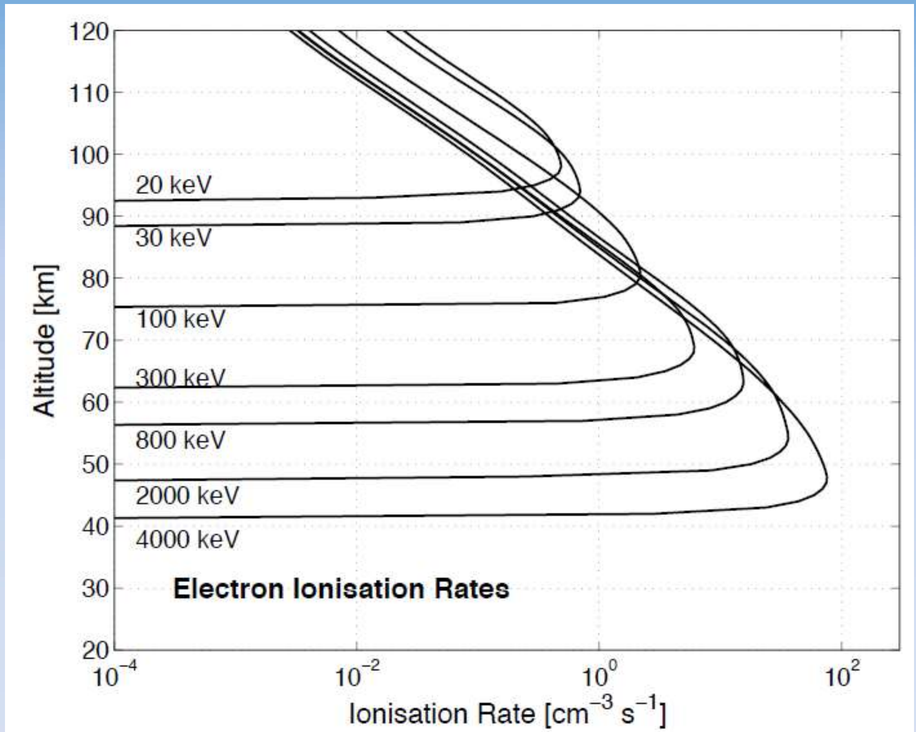


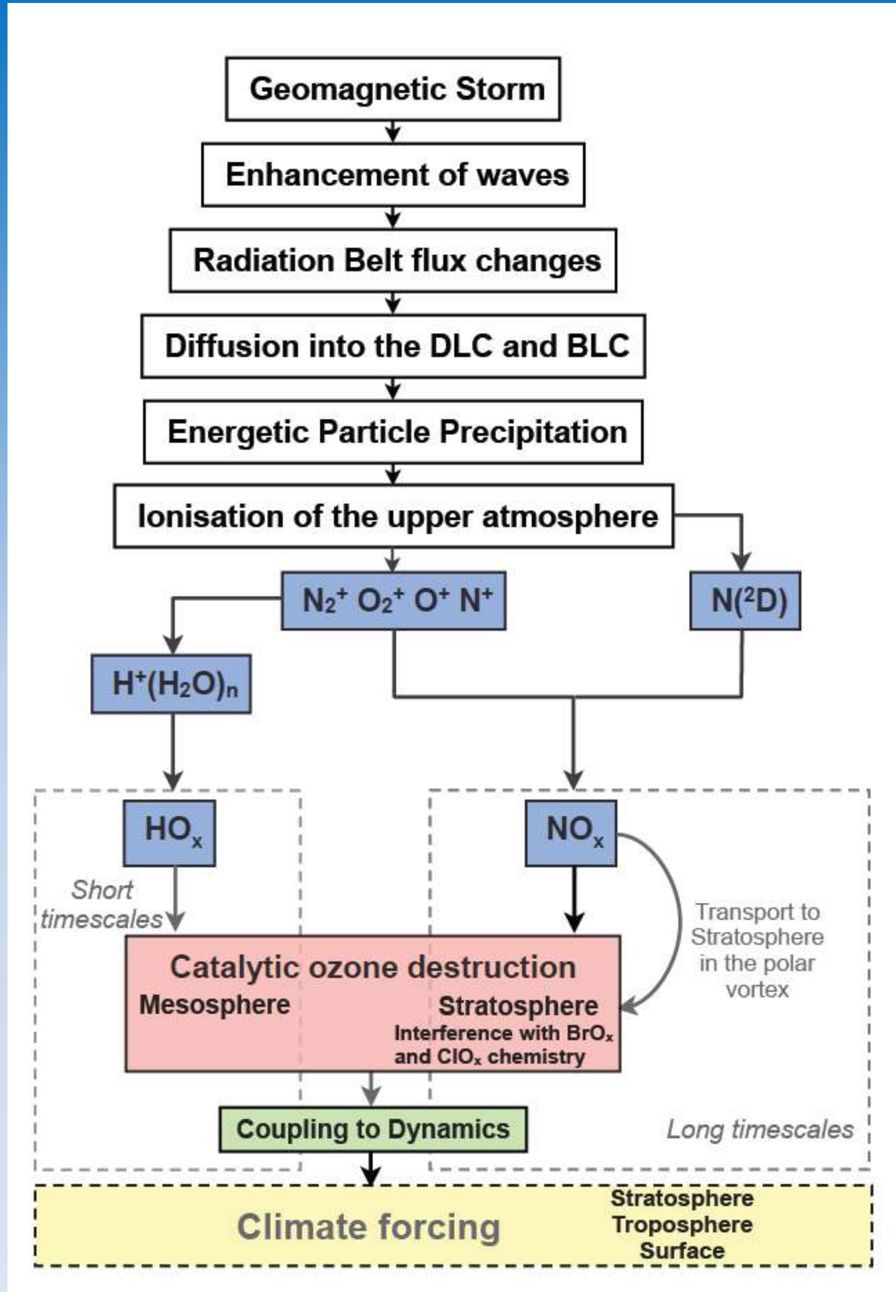
Radiation belt changes during Halloween storms: Baker et al. (*Nature, 2004)*

Internal Charging of Spacecraft



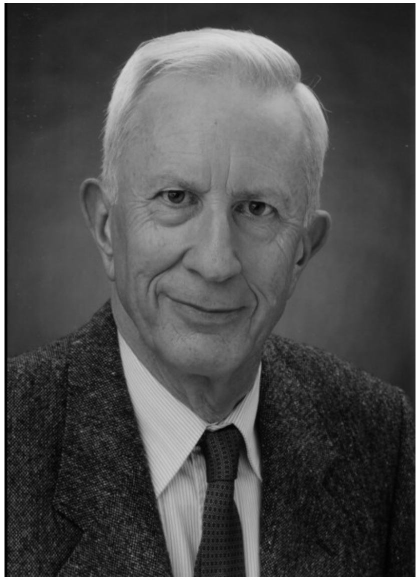
Energetic Electron Precipitation into the Atmosphere



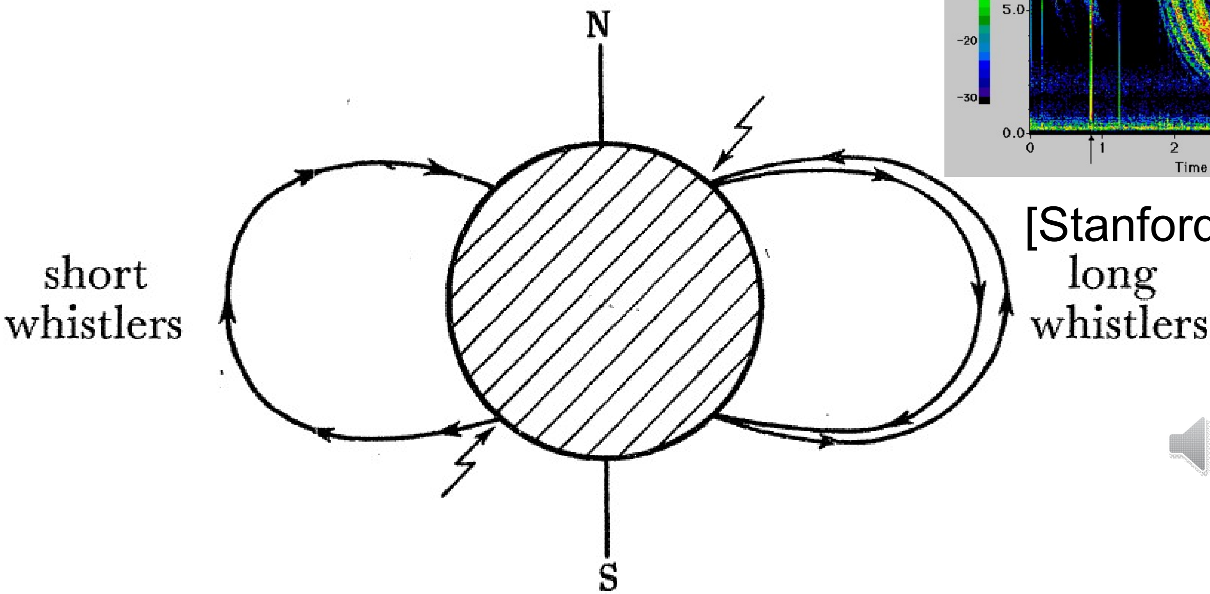


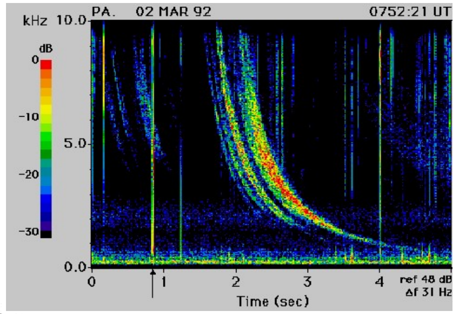
[*Clilverd et al.*, Oxford Univ. Press, 2015]

Whistling Atmospherics



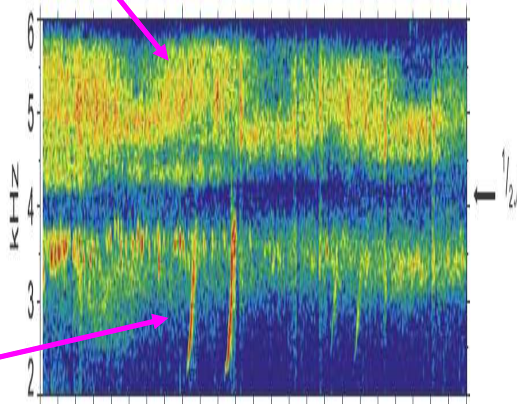
[L. R. O. Storey, 1953]

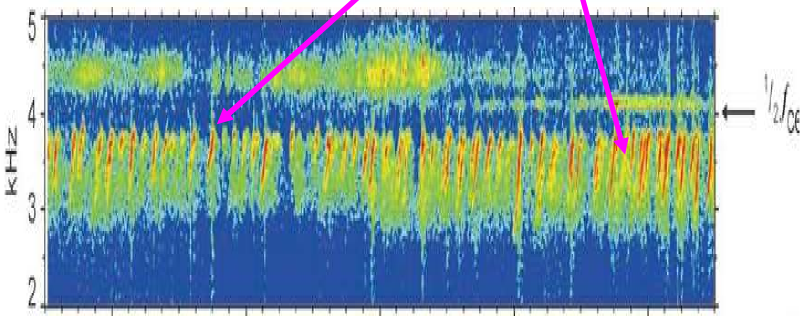




[Stanford Web]

Outer Ionosphere = Magnetosphere

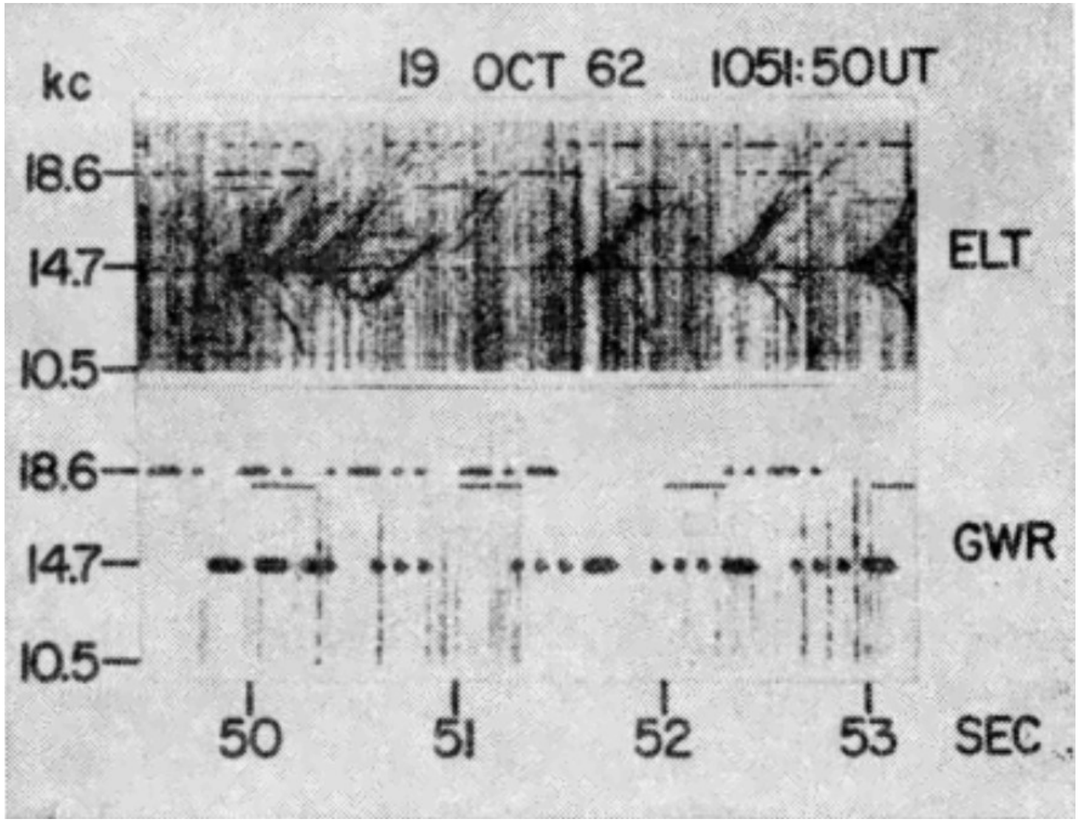




[Santolik et al.,JGR, 2003]

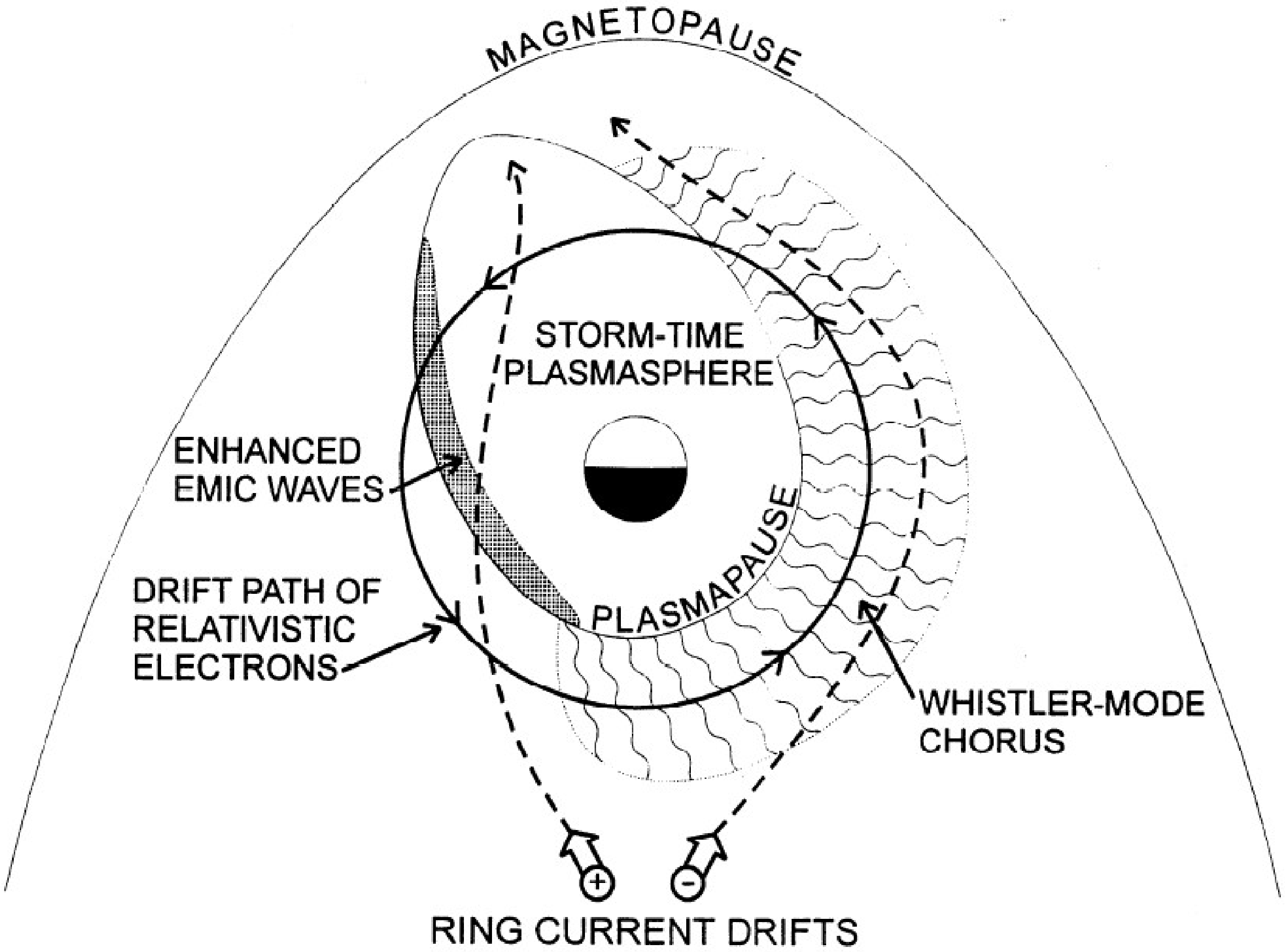
VLF Triggered Emissions

[R. A. Helliwell, et al., JGR, 1964]



Rising and falling tones from the Morse code dashes

Relativistic Theory of Wave-Particle Resonant Diffusion with Application to Electron Acceleration in the Magnetosphere



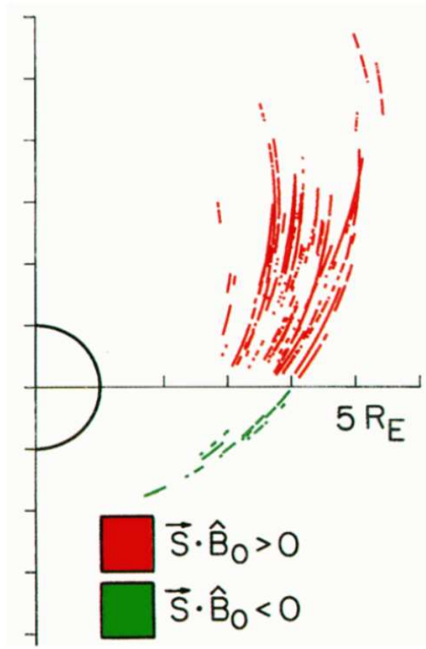
[*Summers et al., JGR, 1998*]

New Generation of Spacecraft Observation

Polar spacecraft

Source very close to Geomagnetic Equator

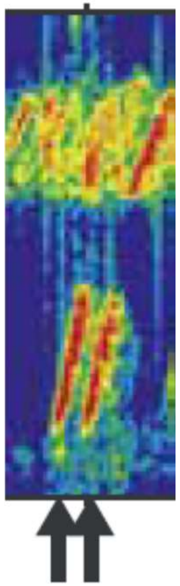
Absolute Instability at the Equator

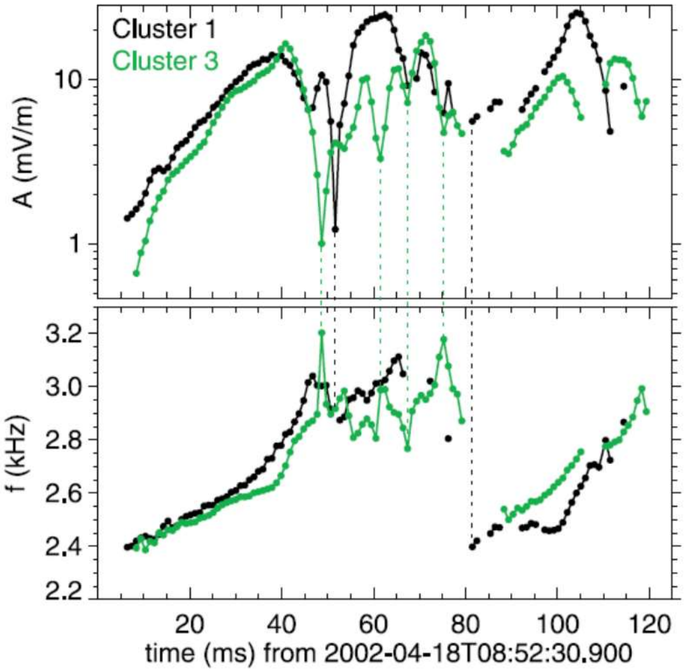


[LeDocq et al., 1998]

Cluster spacecraft

Large Wave Amplitude: 10-100pt





Nonlinear Wave-Particle Interaction

[*Santolik et al., JGR, 2003*]

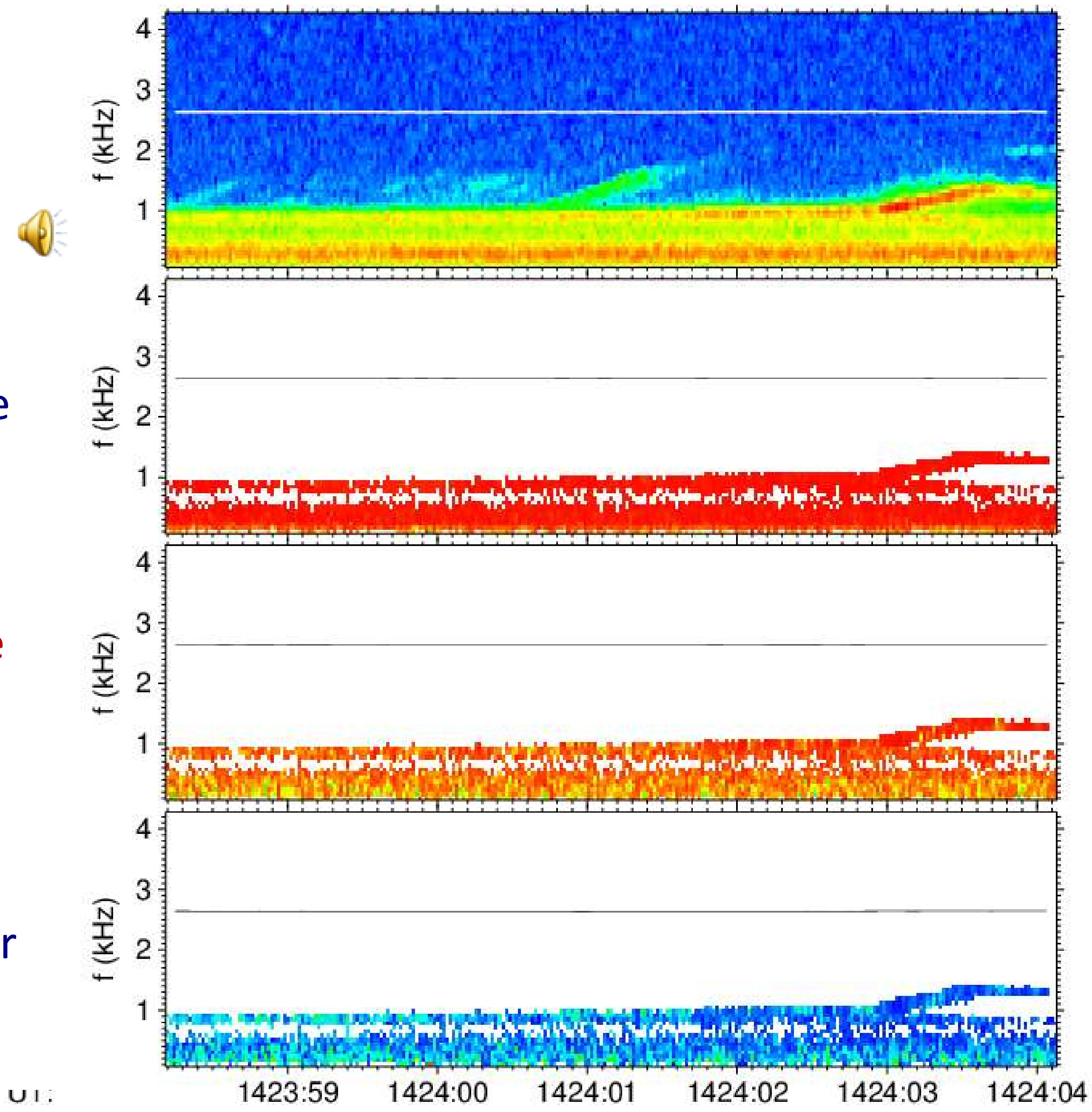
EMFISIS Waves, Van Allen Probe B, 2 July 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



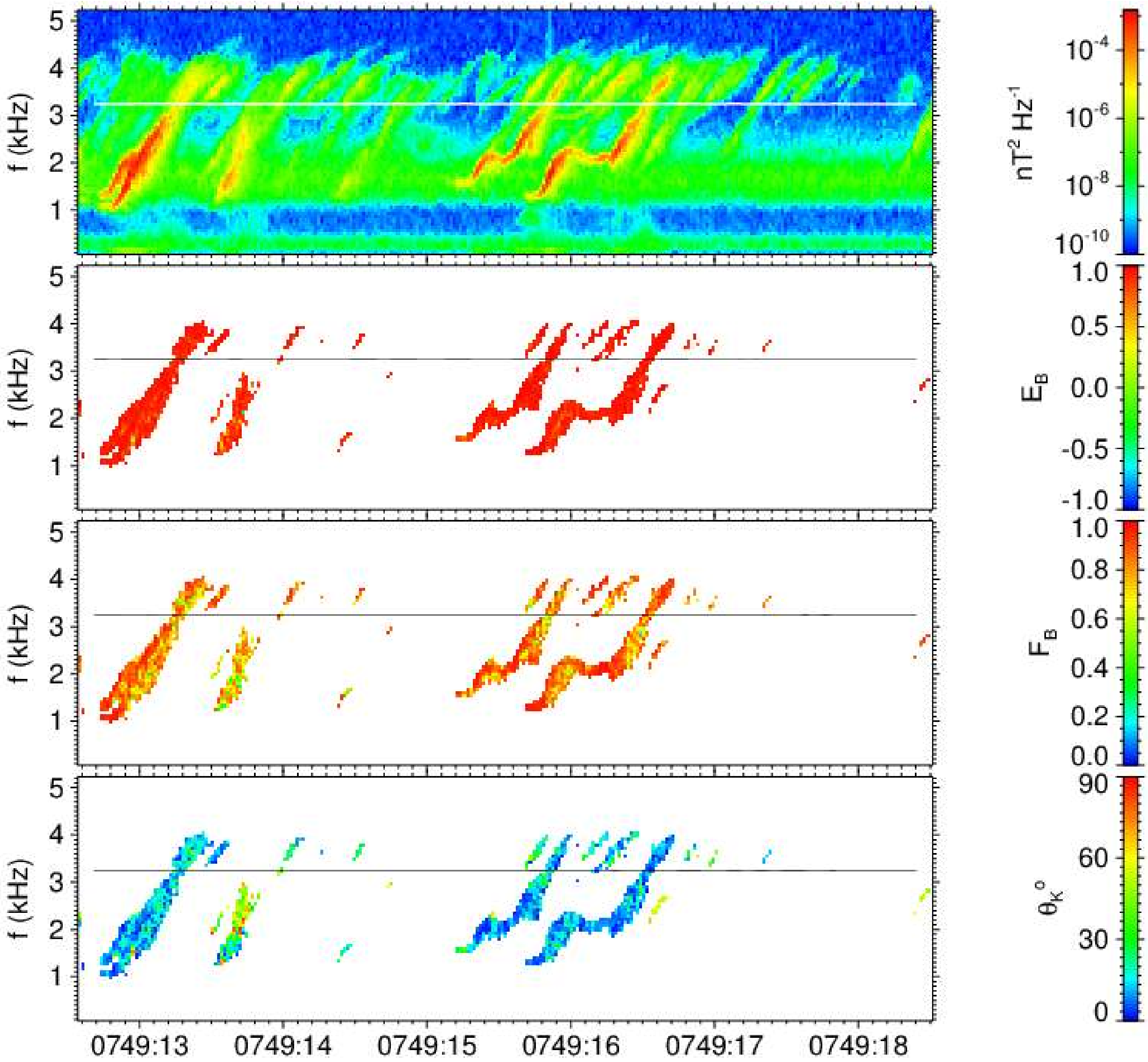
EMFISIS Waves, Van Allen Probe A, 14 April 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



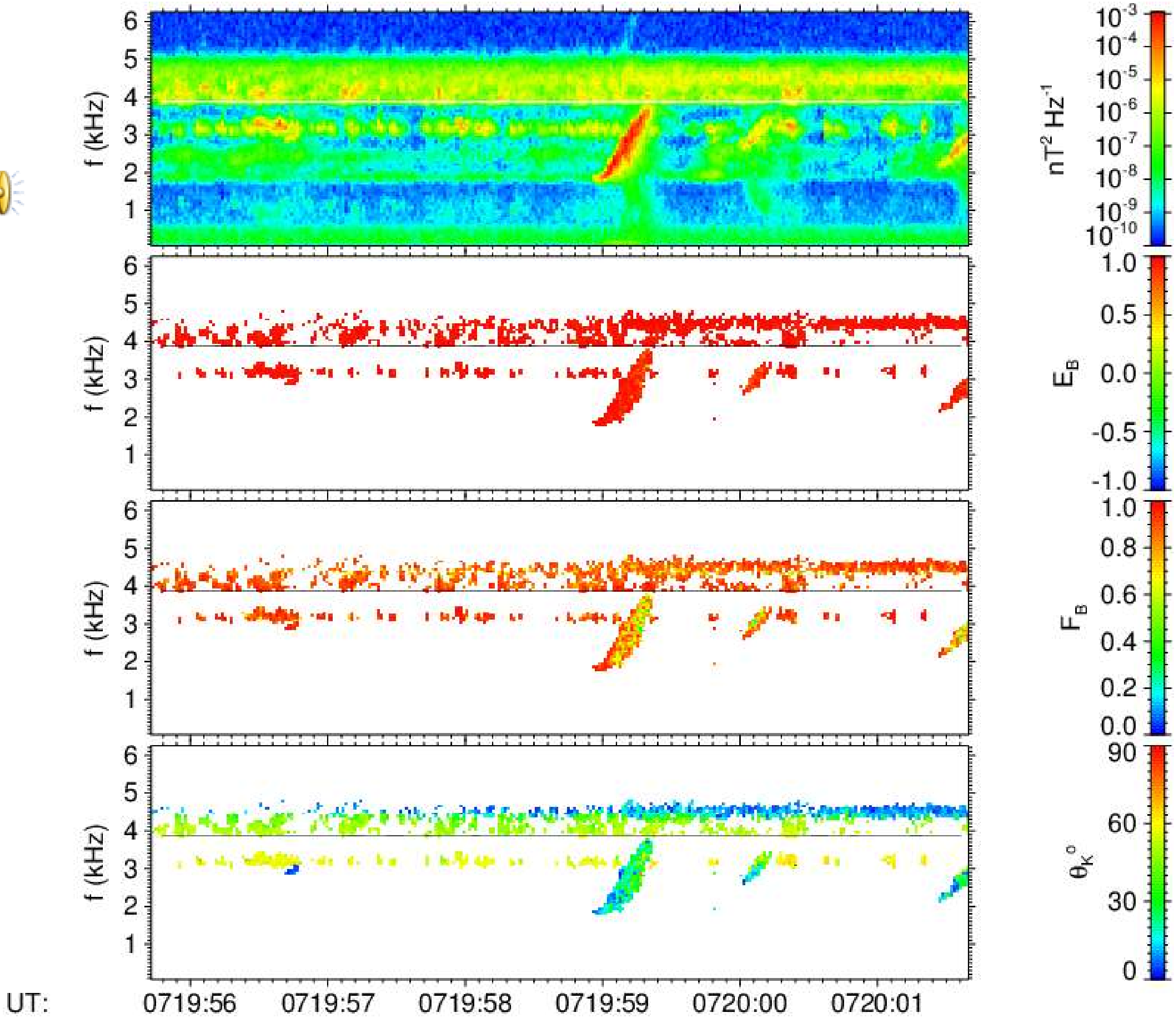
EMFISIS Waves, Van Allen Probe A, 8 June 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



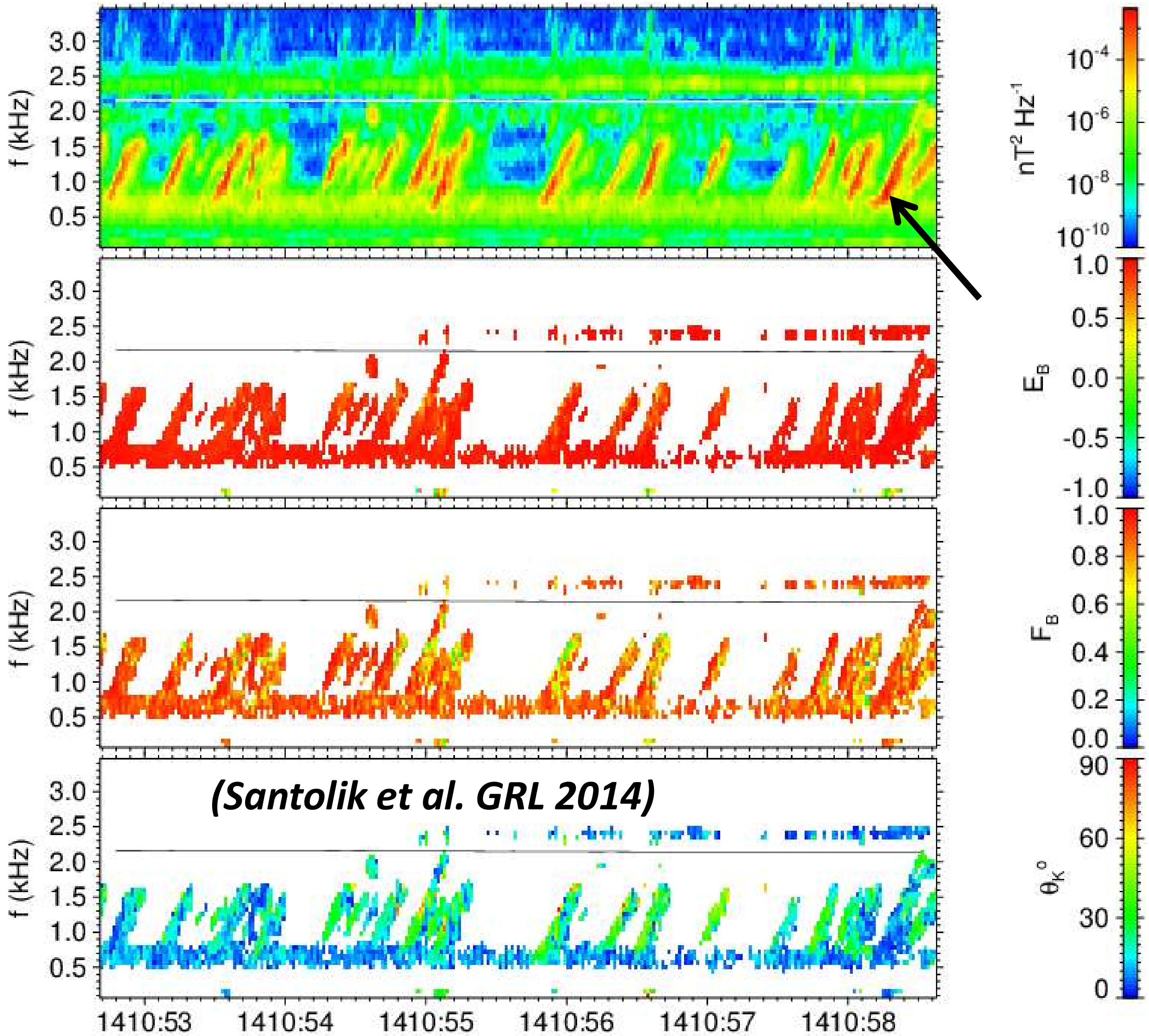
EMFISIS Waves, Van Allen Probe A, 14 Nov 2012

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

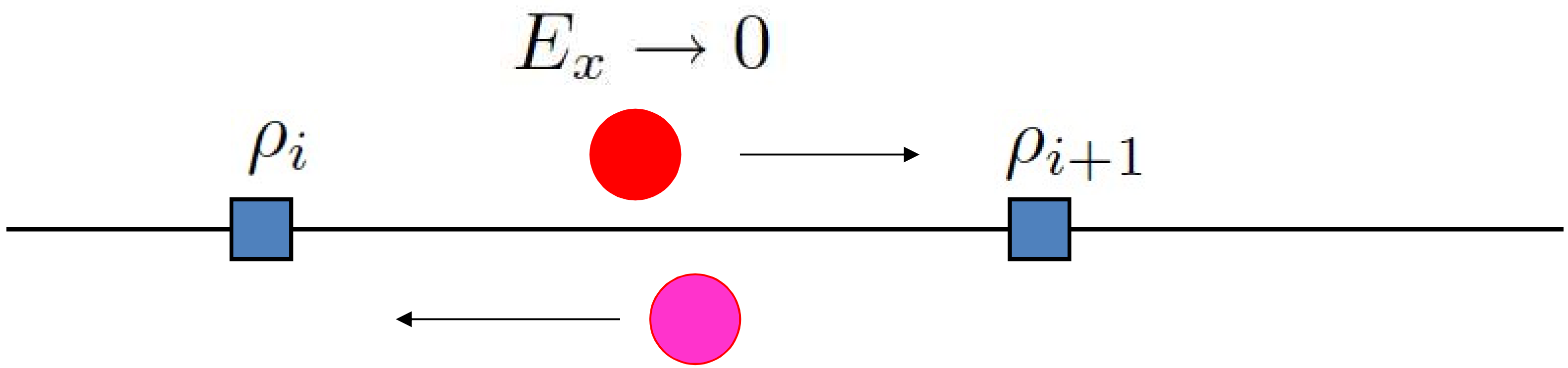
angle between the wave vector and the background magnetic field



PIC code for Space Plasmas

- **Space Plasmas**: **Collisionless**
- **Particle-In-Cell Code**
- **Particles**: x(t), **v**(t)
- **Fields**: ***E***(t, X), ***B***(t, X)

E and ***B*** are defined on grid points, and calculated from ρ and ***J***. The electrostatic force between two particles in the same cell disappears.



Maxwell's Equations

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

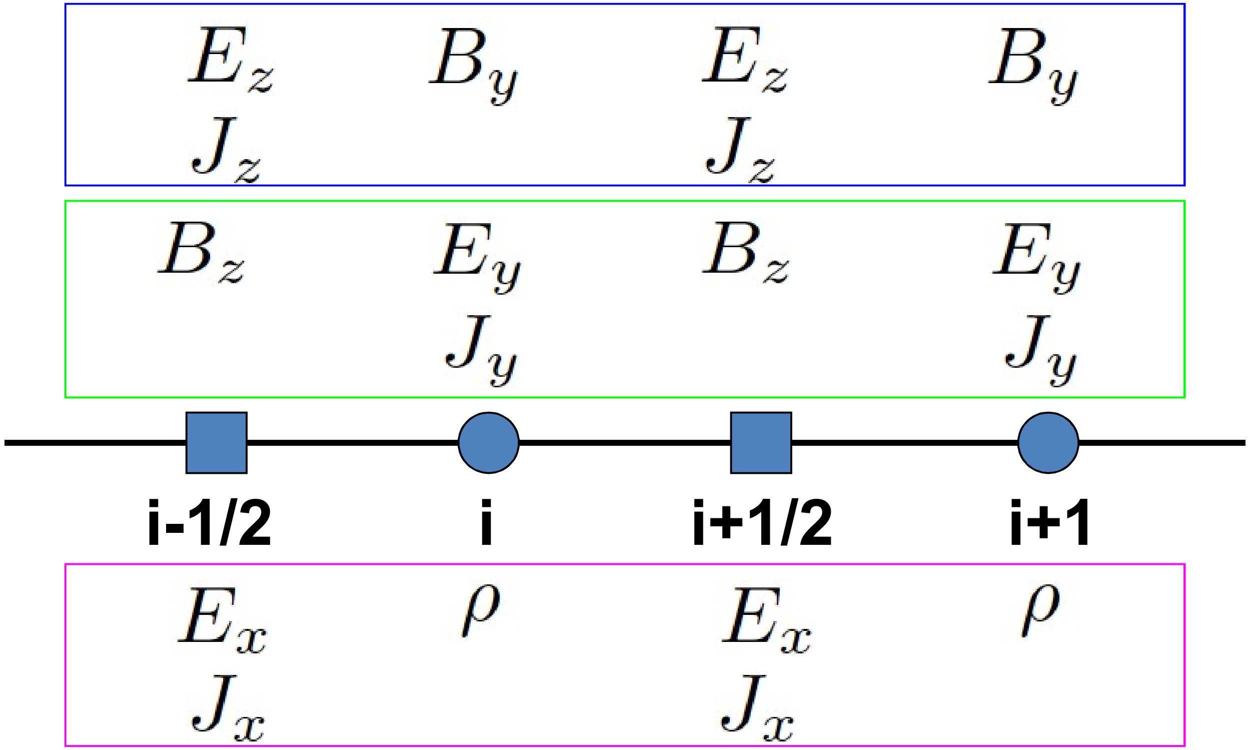
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

where $\varepsilon_0 \mu_0 = \dfrac{1}{c^2}$

Grid Assignment



$$\frac{\partial E_z}{\partial t} = c^2 \frac{\partial B_y}{\partial x} - J_z$$

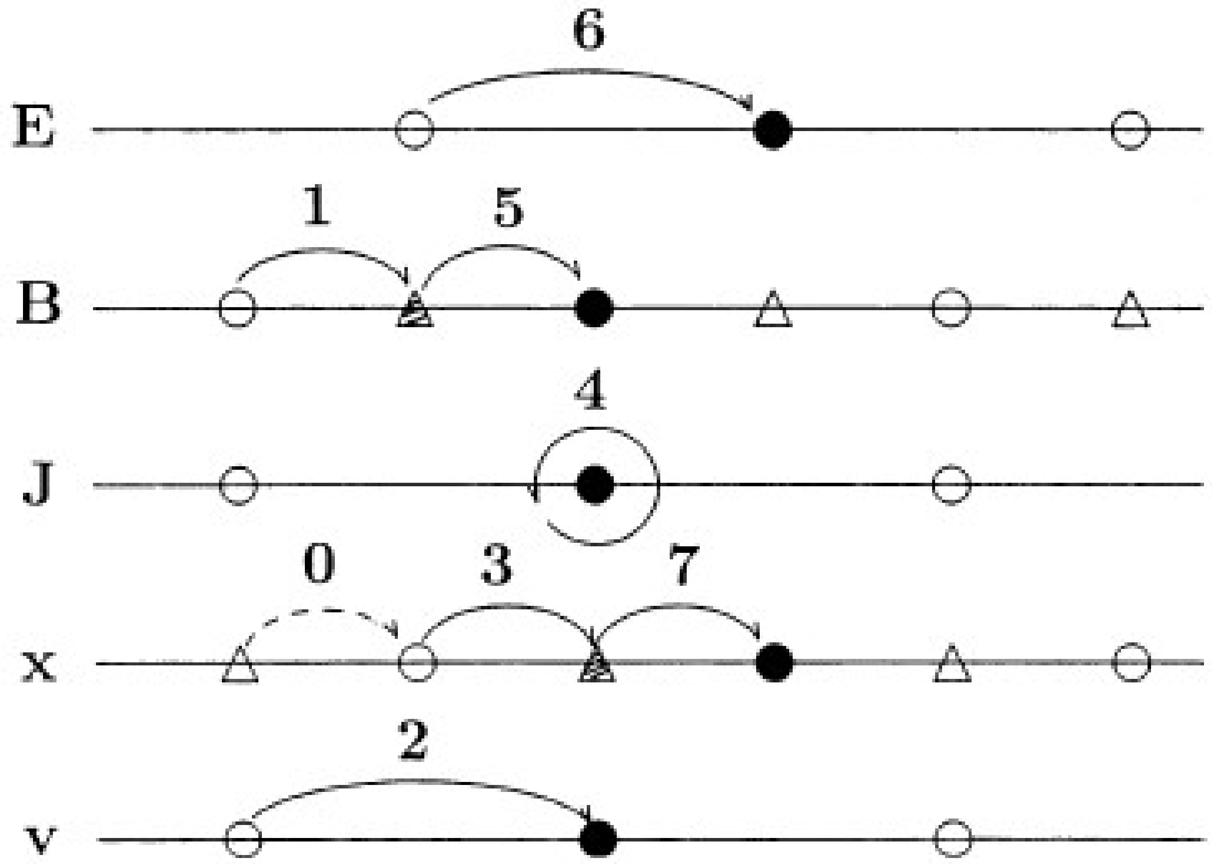
$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$\frac{\partial E_y}{\partial t} = -c^2 \frac{\partial B_z}{\partial x} - J_y$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial E_x}{\partial t} = -J_x$$

Time Step Chart



Centered Difference Scheme

$$E(X_i, t) = E_o \exp(ikX_i - i\omega t)$$

$$\frac{\partial E(X_i,t)}{\partial x} = \frac{E(X_i + \Delta x/2, t) - E(X_i - \Delta x/2, t)}{\Delta x}$$

$$= \frac{1}{\Delta x}\left[\exp(ik\Delta x/2) - \exp(-ik\Delta x/2)\right] E(X_i, t)$$

$$= i\frac{\sin(k\Delta x/2)}{\Delta x/2} E(X_i, t) = iKE(X_i, t)$$

$$k \quad \Rightarrow \quad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

$$\omega \quad \Rightarrow \quad \Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}$$

Courant Condition

Electromagnetic modes in vacuum

$$\omega^2 = c^2 k^2$$

Centered Difference Scheme in space and time

$$\Omega^2 = c^2 K^2 \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

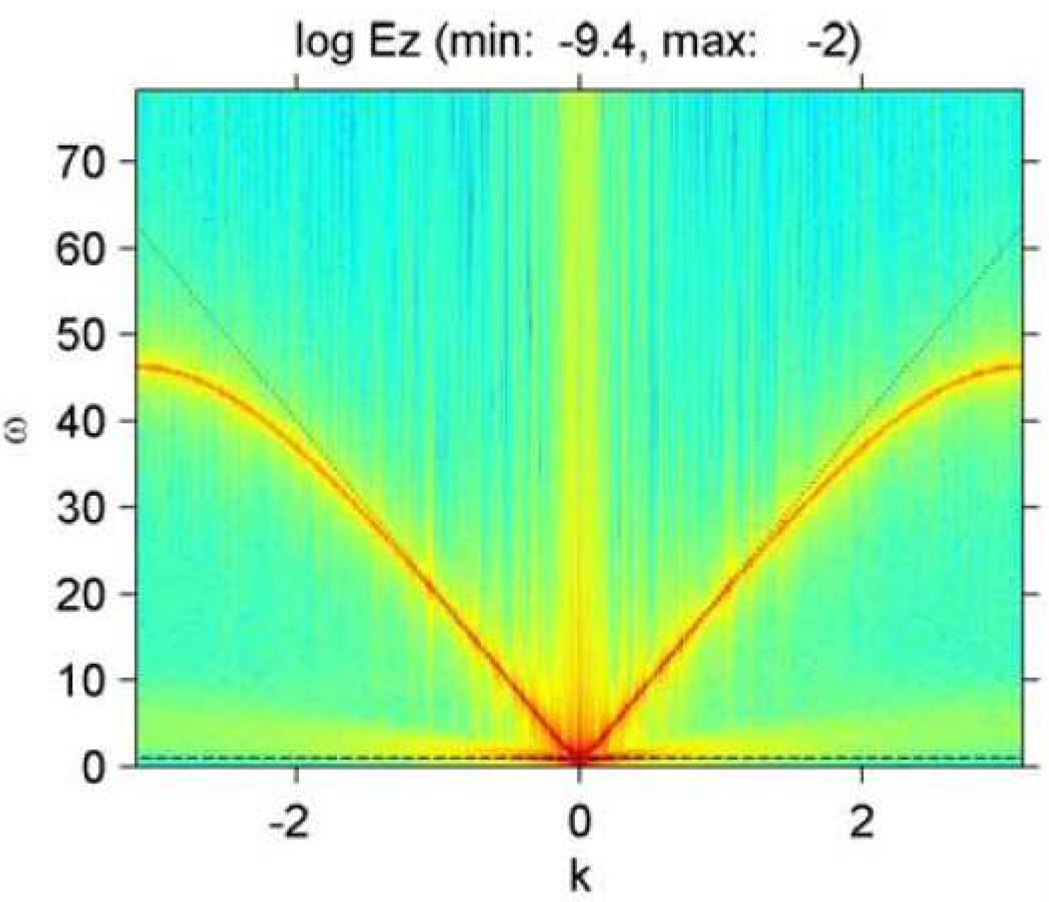
For $k = \dfrac{\pi}{\Delta x}$ we have $\sin(\dfrac{\omega \Delta t}{2}) = \dfrac{\Delta t}{\Delta x} c < 1$

Courant Condition

$$\boxed{c\Delta t < \Delta x}$$

Dispersion Relation of Light Mode

$$\Omega^2 = c^2 K^2$$



$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k \Delta x/2)}{\Delta x/2}$$

Charge Density

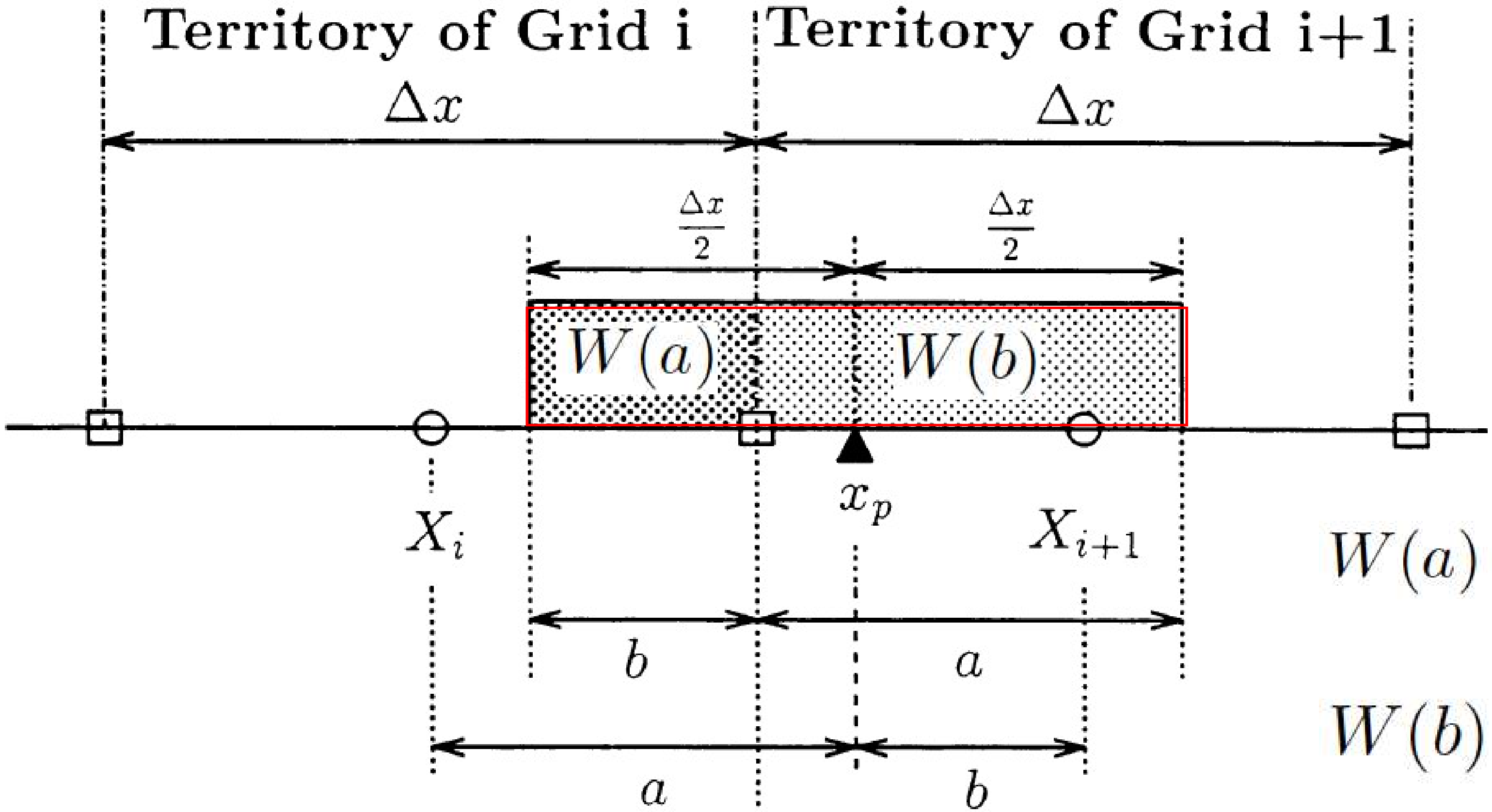
$$\rho_i = \frac{1}{\Delta x} \sum_j^{N_p} q_j W(x_j - X_i)$$

Np: Number of Particles

Shape Function

$$W(x) = 1 - \frac{|x|}{\Delta x}, \quad |x| \le \Delta x$$
$$= 0, \quad |x| > \Delta x$$

"fat particle"



$$W(a) = \frac{b}{\Delta x}$$

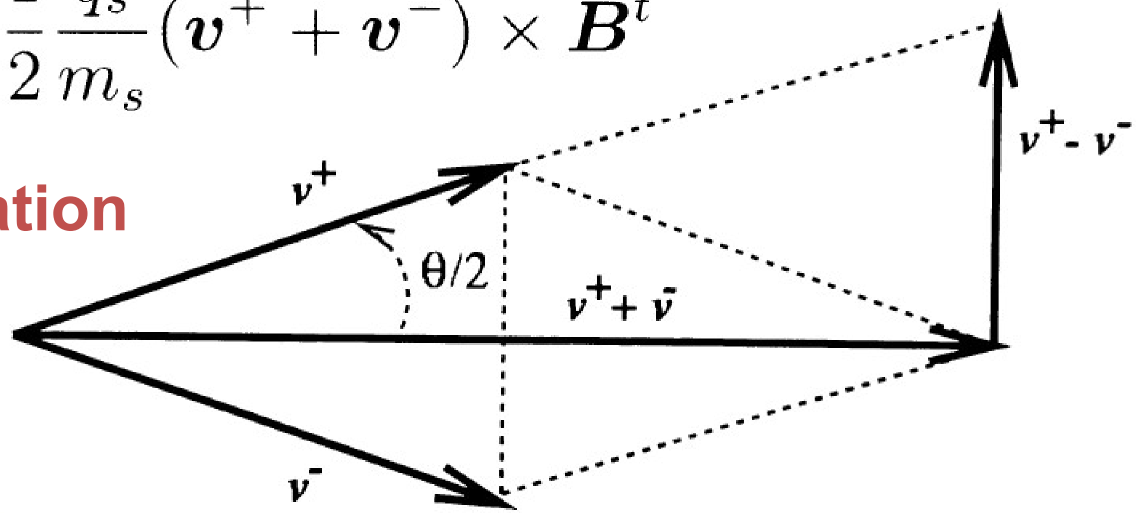
$$W(b) = \frac{a}{\Delta x}$$

Buneman-Boris Method

$$\frac{\boldsymbol{v}^{t+\Delta t/2} - \boldsymbol{v}^{t-\Delta t/2}}{\Delta t} = \frac{q_s}{m_s}\left(\boldsymbol{E}^t + \frac{\boldsymbol{v}^{t+\Delta t/2} + \boldsymbol{v}^{t-\Delta/2}}{2} \times \boldsymbol{B}^t\right)$$

$$\boldsymbol{v}^- = \boldsymbol{v}^{t-\Delta t/2} + \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2} \qquad \boldsymbol{v}^+ = \boldsymbol{v}^{t+\Delta t/2} - \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2}$$

$$\frac{\boldsymbol{v}^+ - \boldsymbol{v}^-}{\Delta t} = \frac{1}{2}\frac{q_s}{m_s}(\boldsymbol{v}^+ + \boldsymbol{v}^-) \times \boldsymbol{B}^t$$



Kinetic Energy Conservation

$$(\boldsymbol{v}^+)^2 = (\boldsymbol{v}^-)^2$$

Small Phase Delay

$$\Omega_c = \frac{\tan^{-1}\omega_c\Delta t/2}{\Delta t/2}$$

$$\Omega_c/\omega_c = 0.9967 \text{ with } \omega_c\Delta t = 0.2$$

Relativistic Equation of Motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\boldsymbol{v}) = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\boldsymbol{u} = \frac{c}{\sqrt{c^2 - |\boldsymbol{v}|^2}}\boldsymbol{v}$$

$$\boldsymbol{B}_u = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{B}$$

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}_{\boldsymbol{u}}\right)$$

$$\boldsymbol{v} = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{u}$$

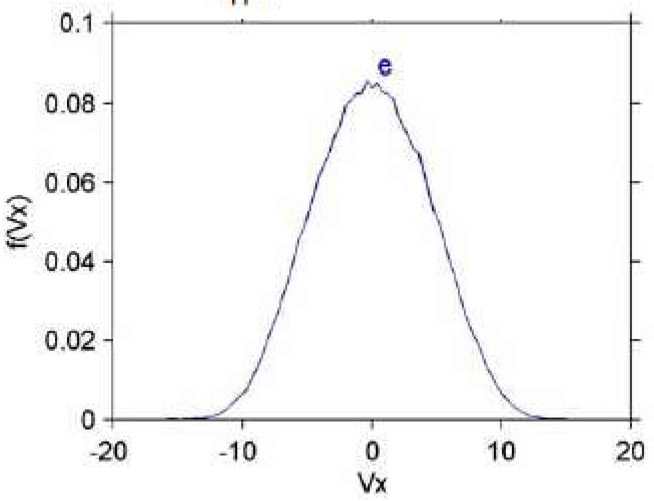
Initial Velocity Distribution Function

$$f(u_{\|}, u_{\perp}) \propto \exp\left(-\frac{(u_{\|} - V_{d\|})^2}{2V_{t\|}^2} - \frac{(u_{\perp} - V_{d\perp})^2}{2V_{t\perp}^2}\right)$$

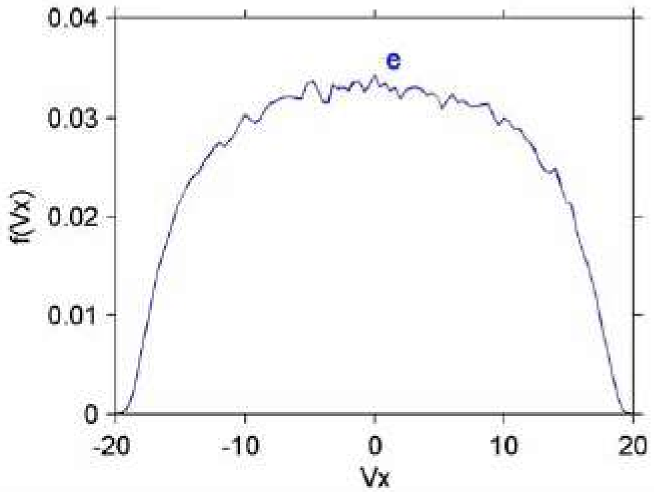
$$\boldsymbol{v} = \boldsymbol{u}/\gamma = \frac{c}{\sqrt{c^2 + u_x^2 + u_y^2 + u_z^2}}\boldsymbol{u}$$

$$V_{d\|} = V_{d\perp} = 0$$

$$V_{t\|}/c = 0.25$$



$$V_{t\|}/c = 1$$

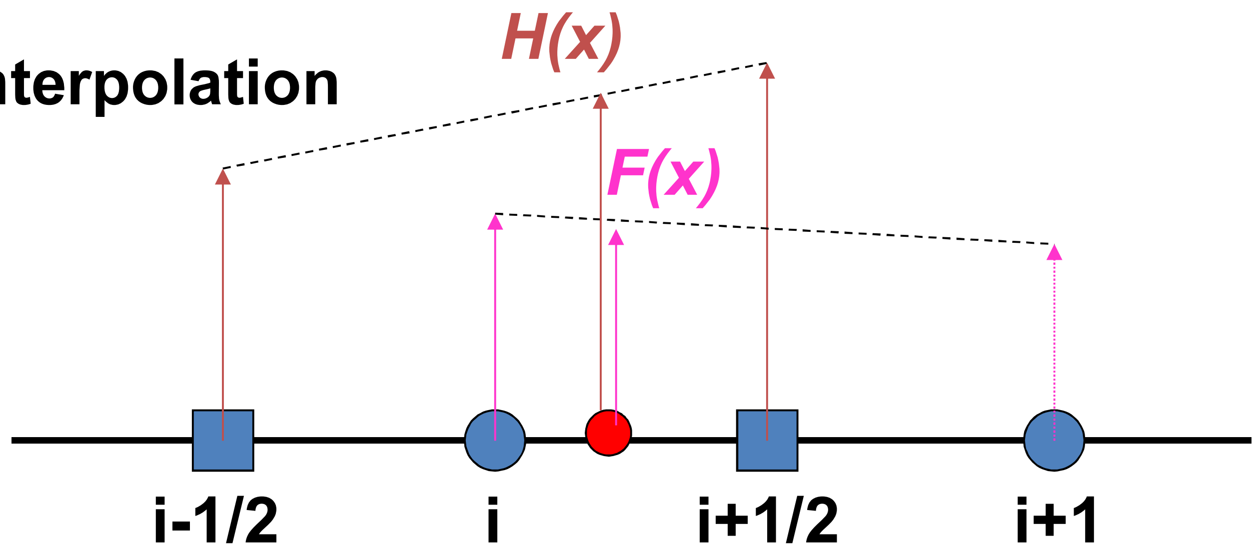


Field Interpolation to Particle Position

$$F(x) = \sum_{i=1}^{N_x} F_i W(x - X_i)$$

$$H(x) = \sum_{i=1}^{N_x} H_{i+1/2} W(x - X_{i+1/2})$$

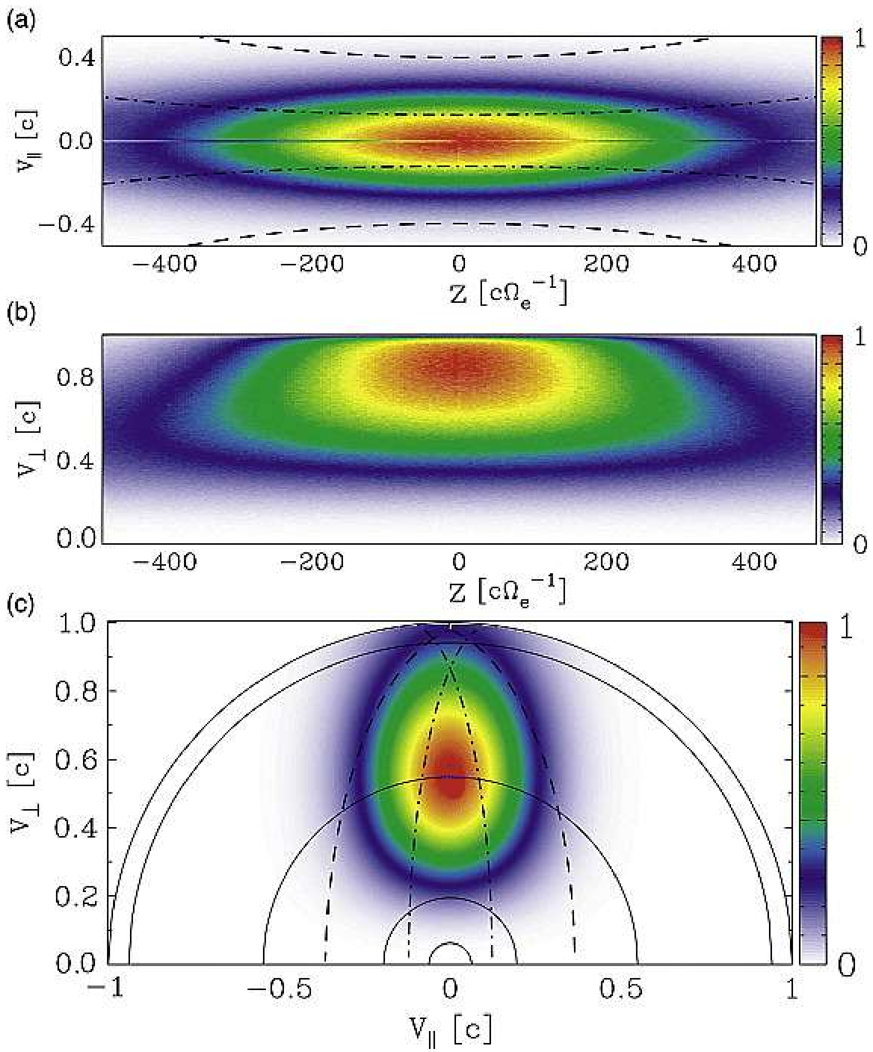
Linear Interpolation

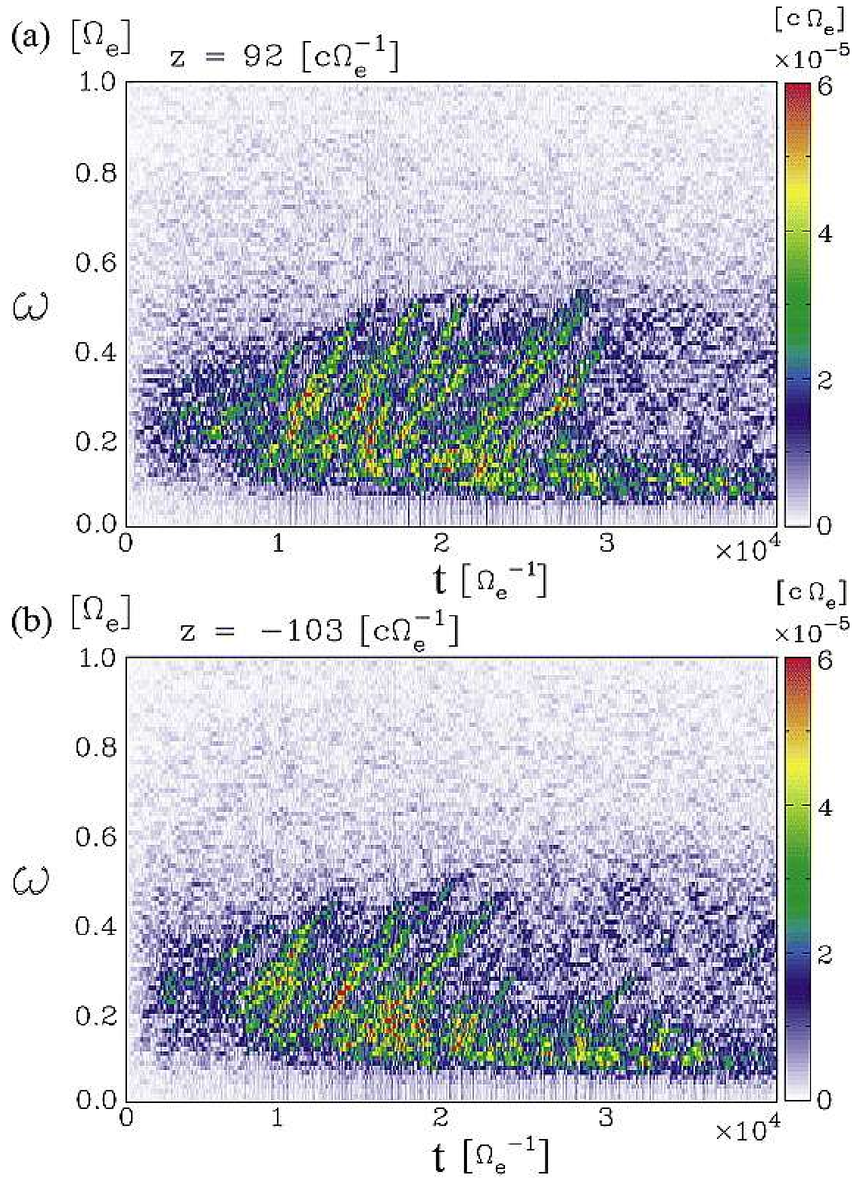


Electron Hybrid Simulation

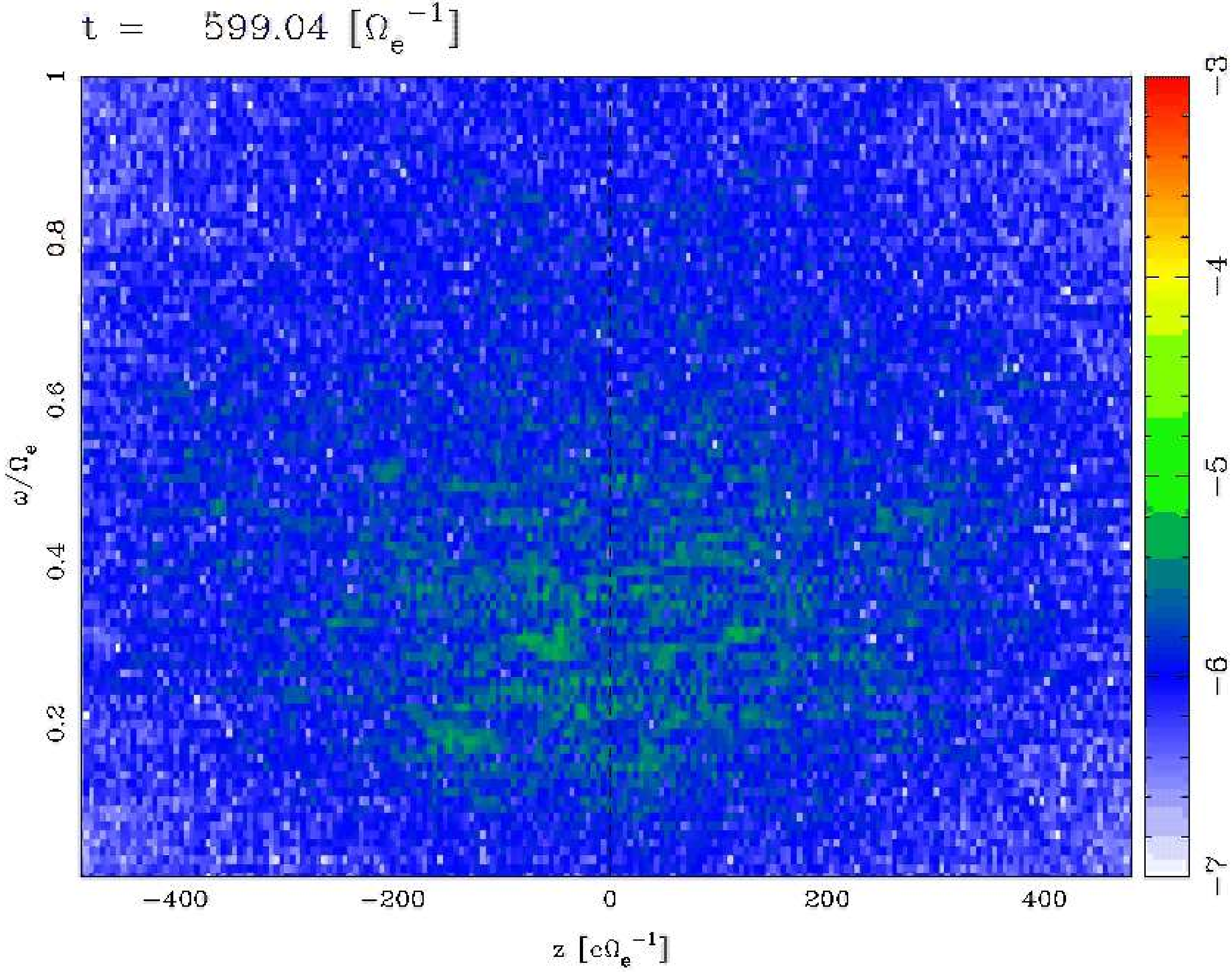
Cold electrons: Fluid

Hot electrons: Particles





[Katoh and Omura,GRL, 2007]

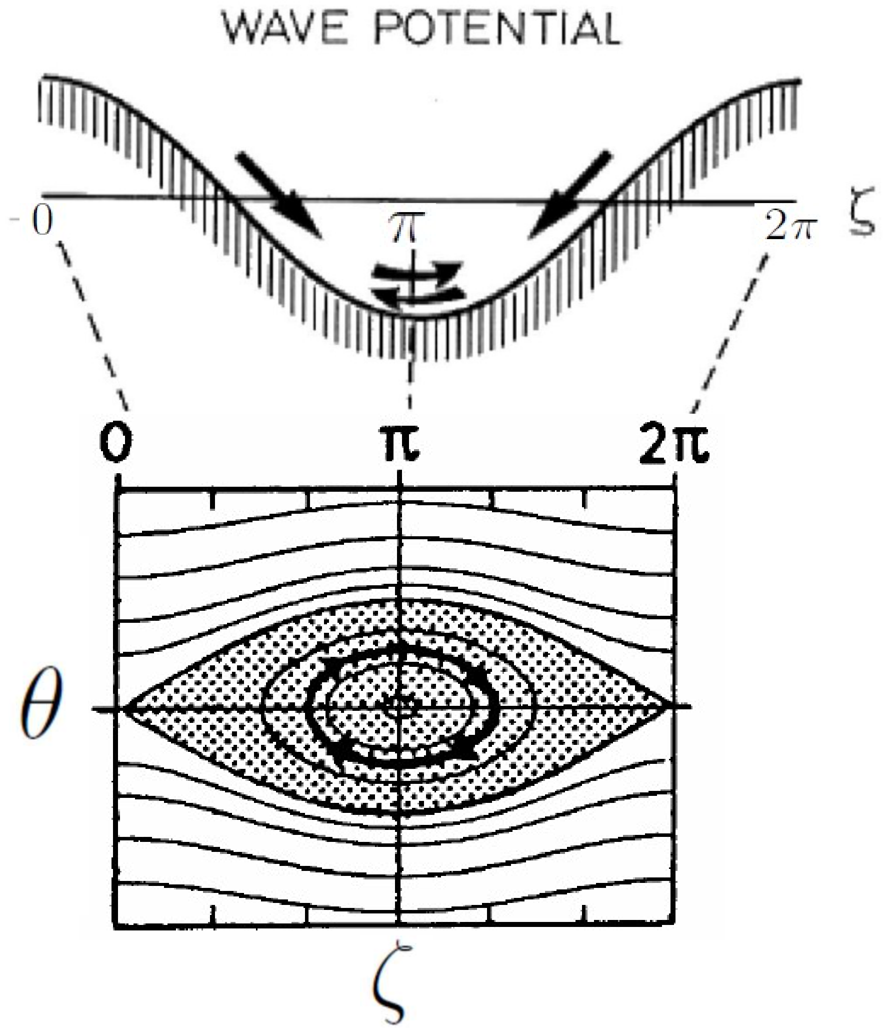


t = 599.04 [Ω_e^{-1}]
ω/Ω_e
1
0.8
0.6
0.4
0.2
−400
−200
0
200
400
z [$c\Omega_e^{-1}$]
−3
−4
−5
−6
−7

Equations of Resonant Particles

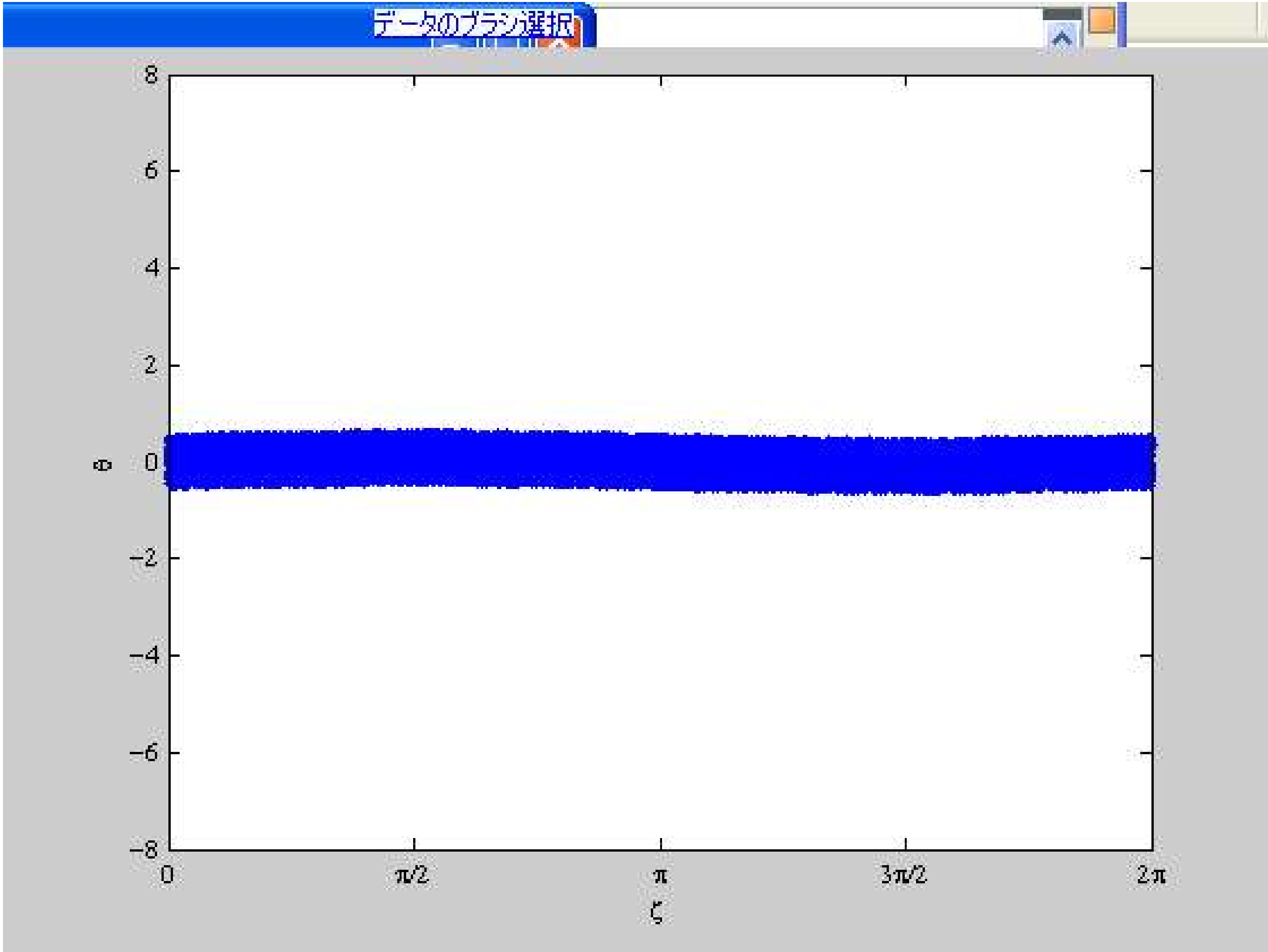
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_t^2 \sin\zeta$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \theta$$



Trapping Frequency

$$\omega_t = \sqrt{\frac{k|q_s|E_w}{m_s}}$$



データのブラシ選択
8
6
4
2
θ
0
−2
−4
−6
−8
0
π/2
π
3π/2
2π
ζ

$$\theta = k(v_{\parallel} - V_p)$$ for Longitudinal Wave

$$\theta = k(v_{\parallel} - V_R)$$ for Whistler-mode Wave

$$V_R = \frac{\omega - \Omega_e}{k}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_t^2(\sin\zeta + \boxed{S})$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \theta$$

S : Inhomogeneity Factor

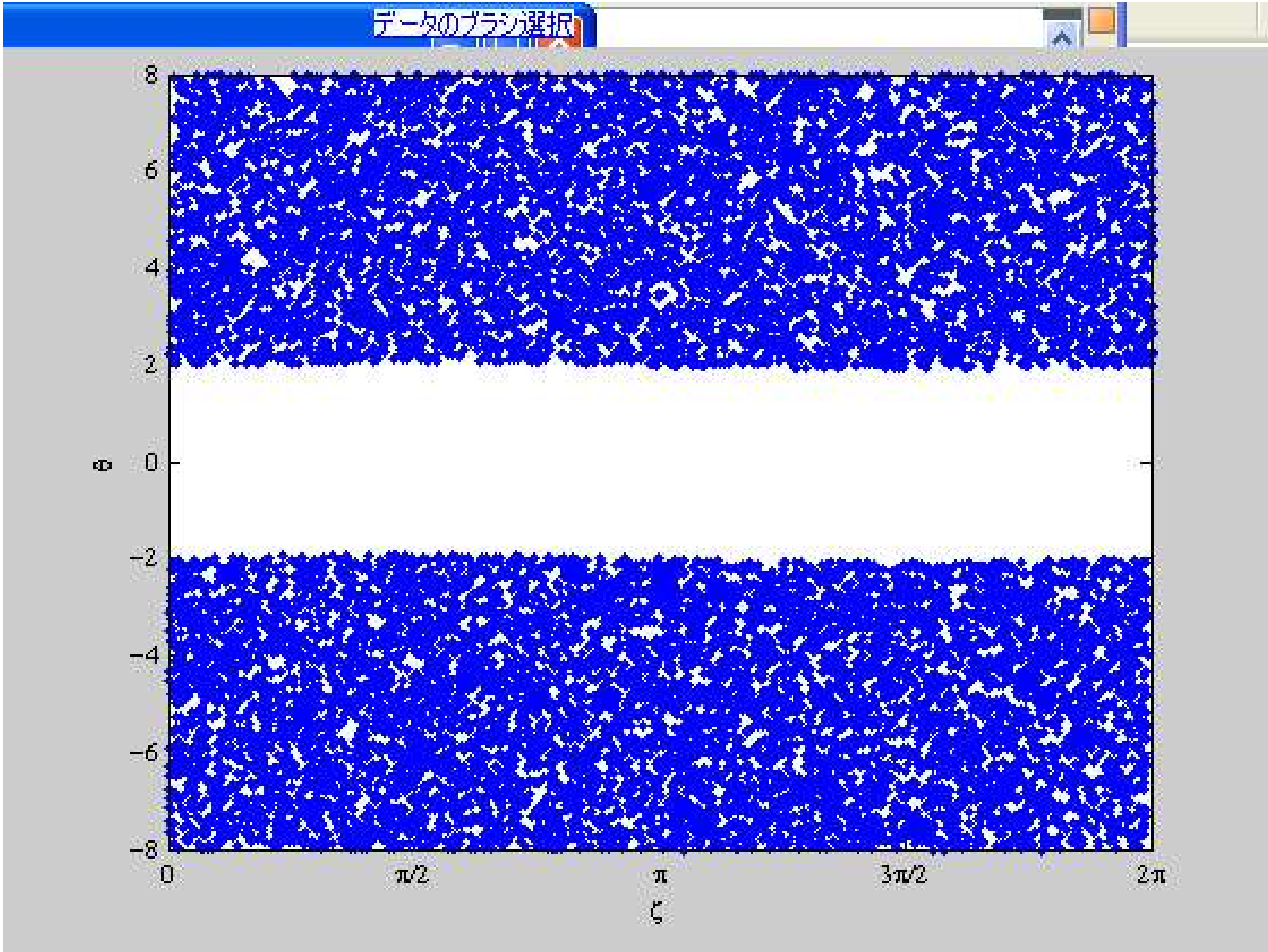
Inhomogeneity Factor

$$S = -\frac{1}{s_0 \omega \Omega_w} \left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h} \right)$$

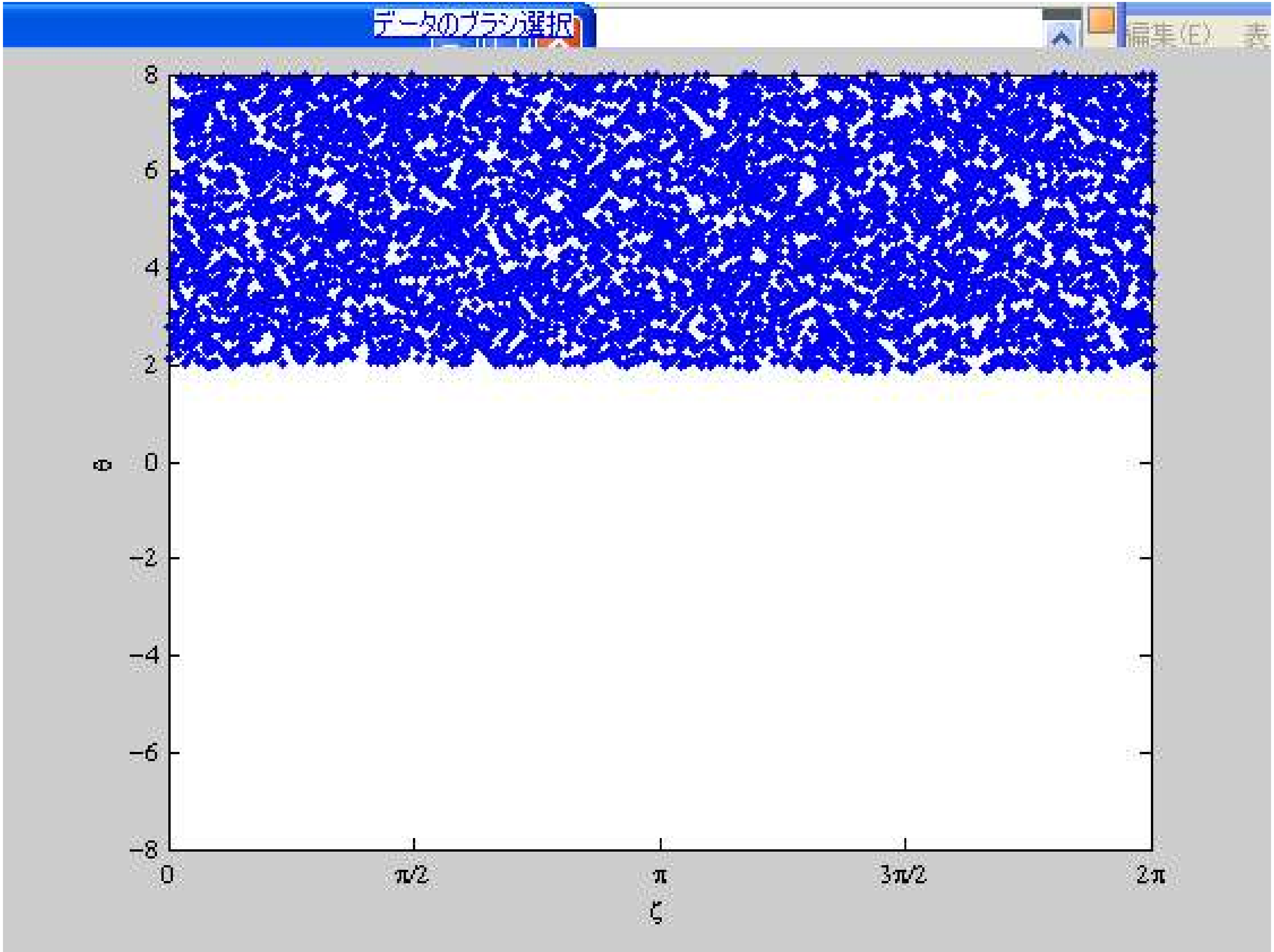
$$s_0 = \frac{\delta}{\xi} \frac{V_{\perp 0}}{c} \qquad s_1 = \gamma \left(1 - \frac{V_R}{V_g}\right)^2$$

$$s_2 = \frac{1}{2\xi\delta} \left\{ \frac{\gamma\omega}{\Omega_e} \left(\frac{V_{\perp 0}}{c} \right)^2 - \left[2 + \Lambda \frac{\delta^2 (\Omega_e - \gamma\omega)}{\Omega_e - \omega} \right] \frac{V_R V_p}{c^2} \right\}$$

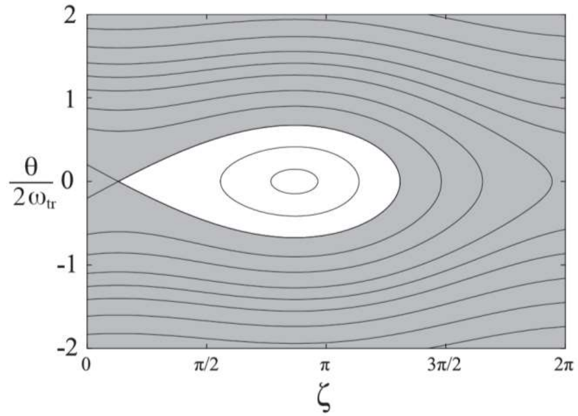
[Omura et al., JGR, 2008; 2009]

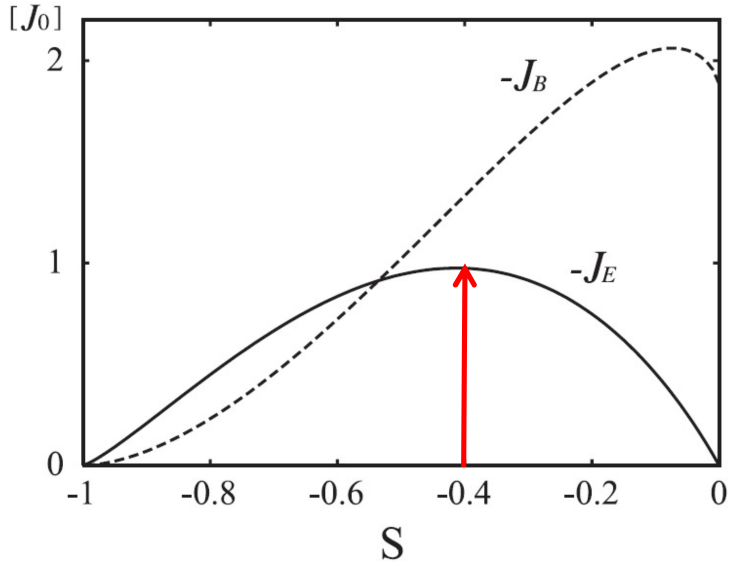


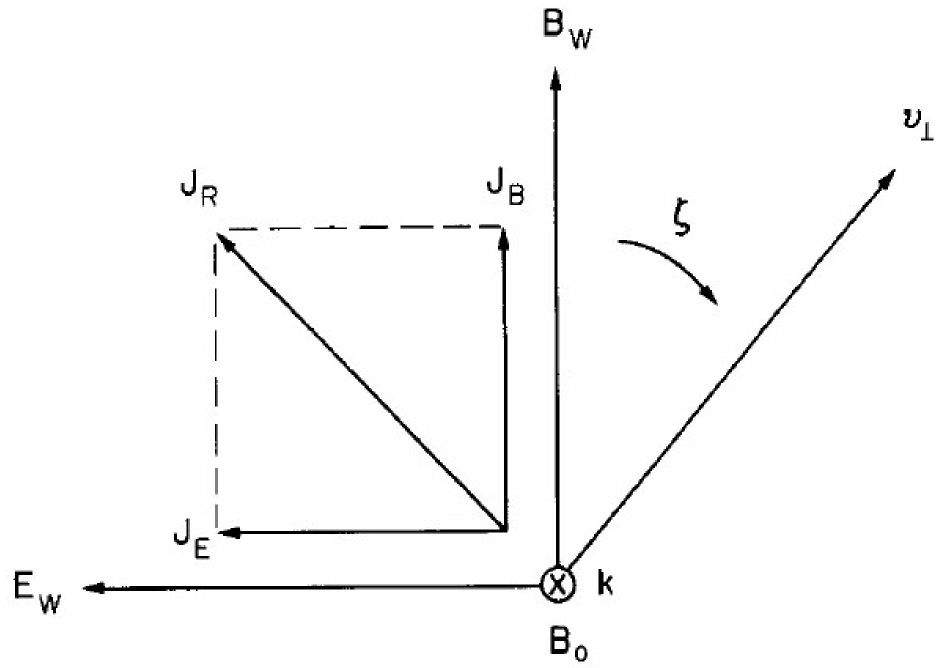
データのブラシ選択
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ


データのブラシ選択
編集(E)
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ

Nonlinear Wave Growth due to Formation of Electromagnetic Electron Hole







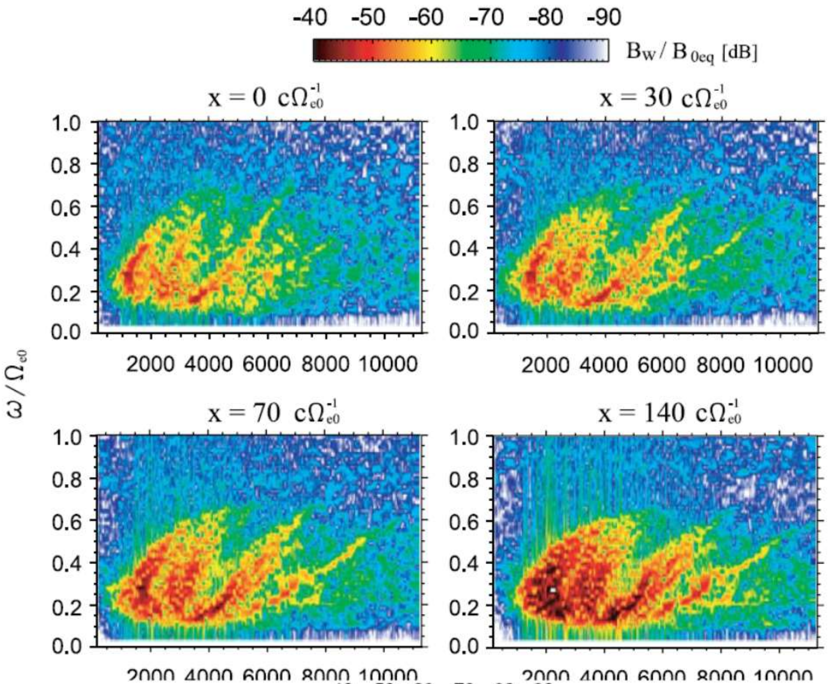
$J_E < 0$: Wave Growth

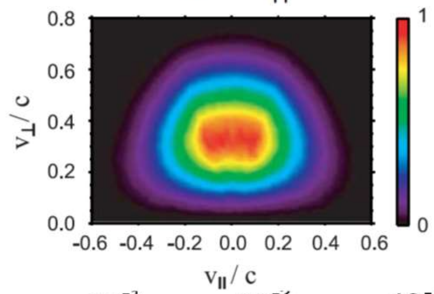
Maximum $-J_E$ $S_{EQ} = -0.4$

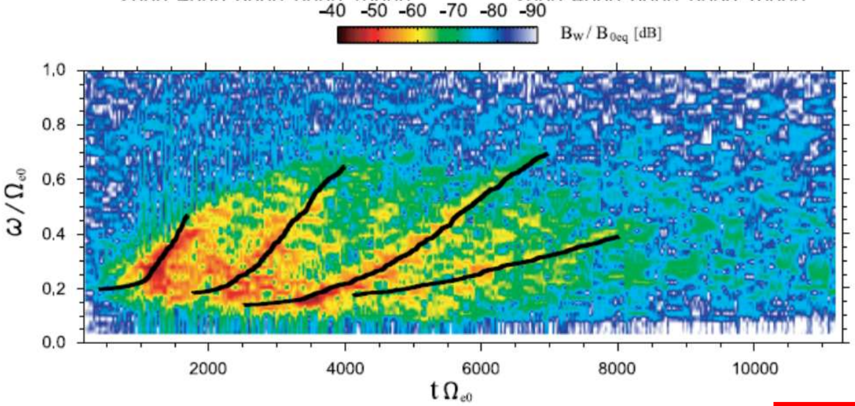
[Omura, Katoh, Summers, JGR, 2008]

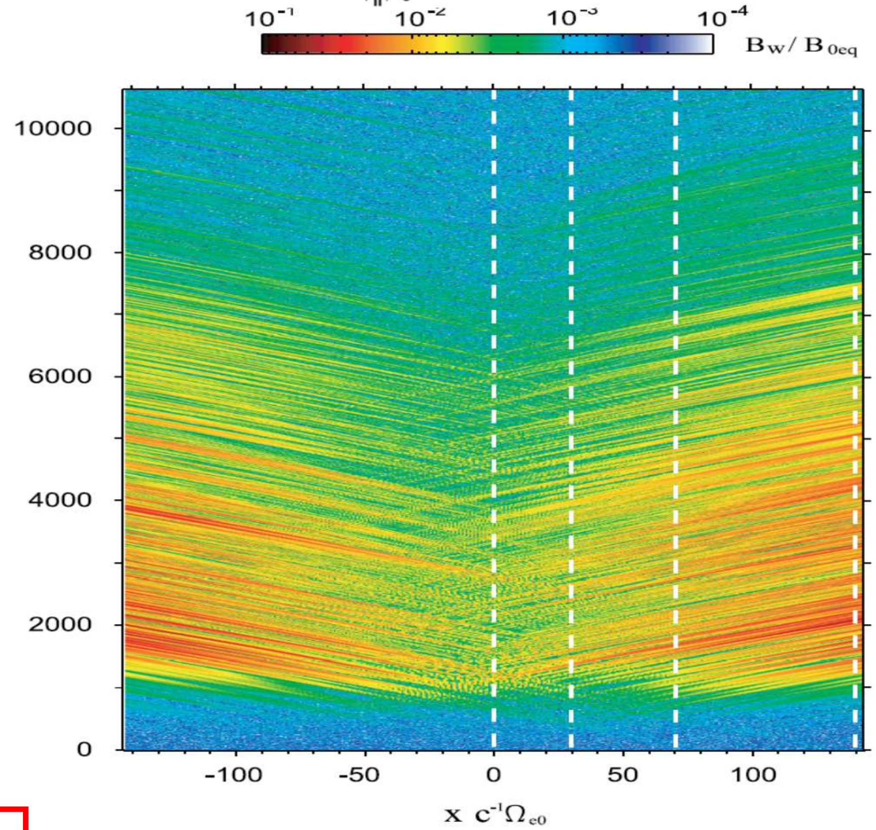
Full-Particle Simulation of Chorus Emission

$$T_\perp / T_\parallel = 3$$





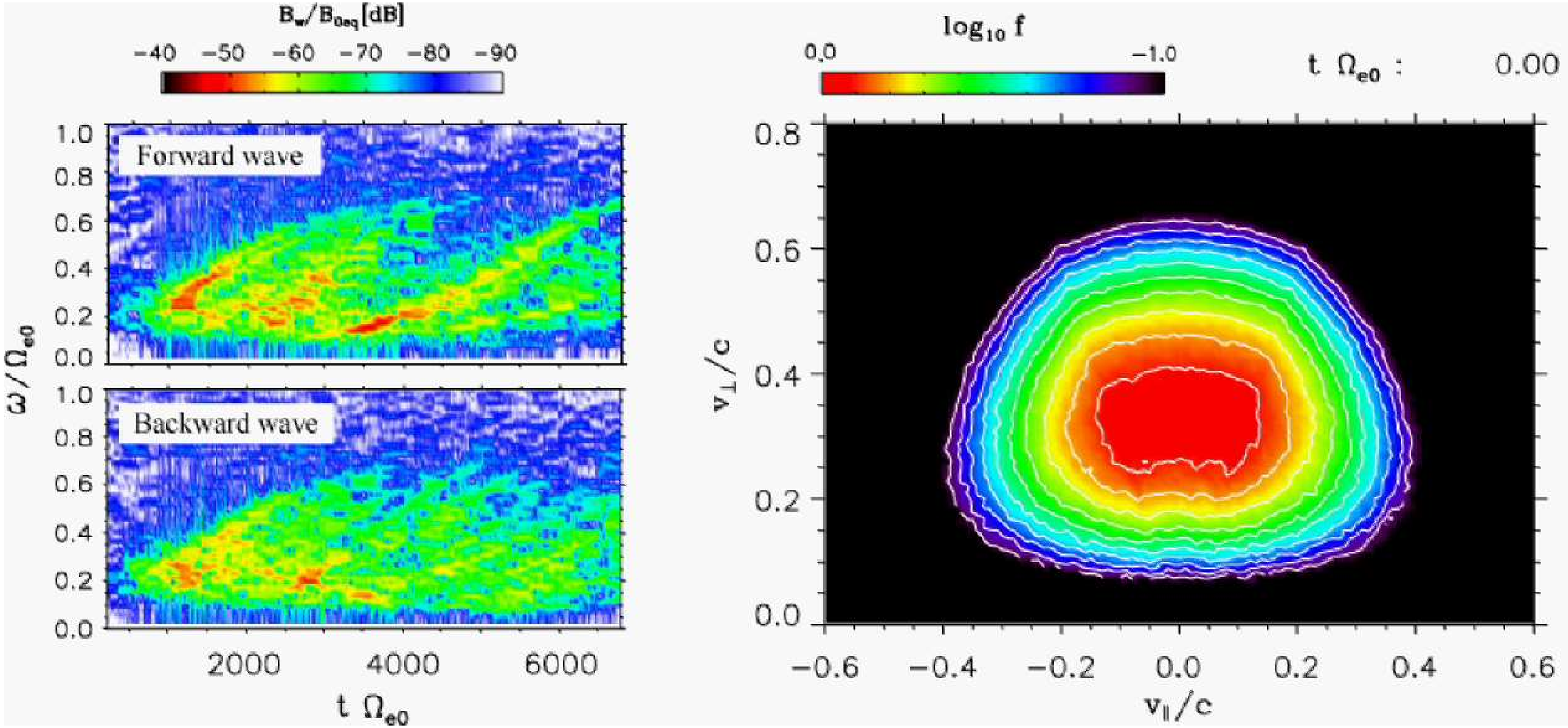




$$\frac{\partial \omega}{\partial t} = \frac{0.4\,\delta}{\gamma\,\xi}\frac{V_{\perp 0}}{c}\frac{\omega}{\Omega_{e0}}\left(1-\frac{V_R}{V_g}\right)^{-2}\frac{B_w}{B_{0eq}}\Omega_{e0}^2$$

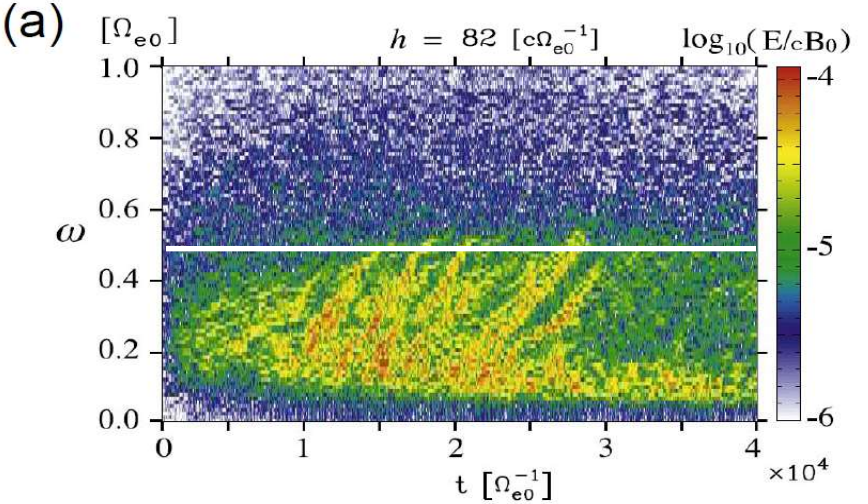
[Hikishima et al., JGR, 2009a]

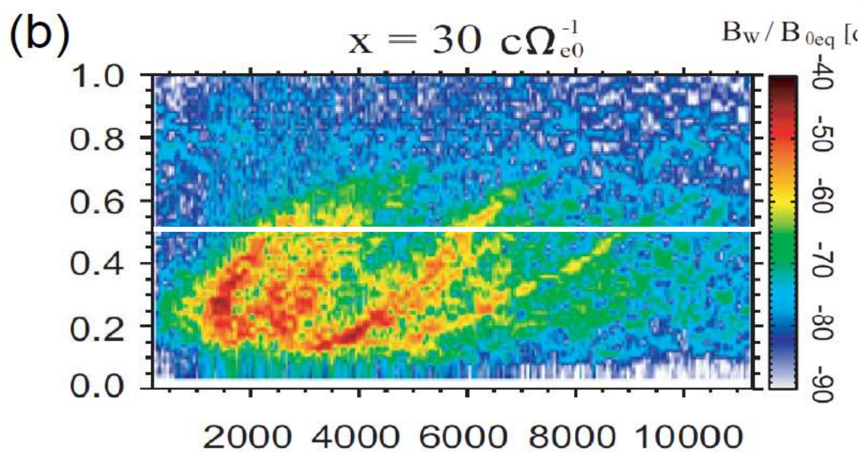
Formation of Pancake Distribution through generation of Chorus emission



[Hikishima et al., JGR, 2009b]

Simulations :
parallel propagation

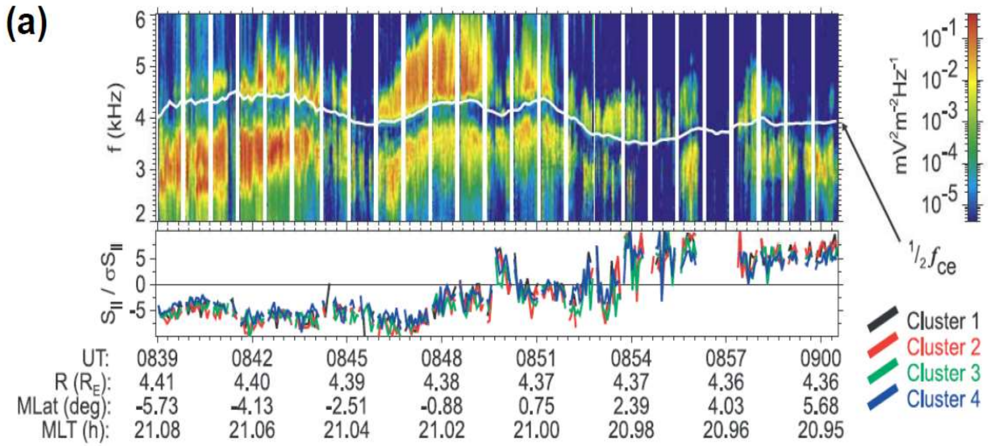


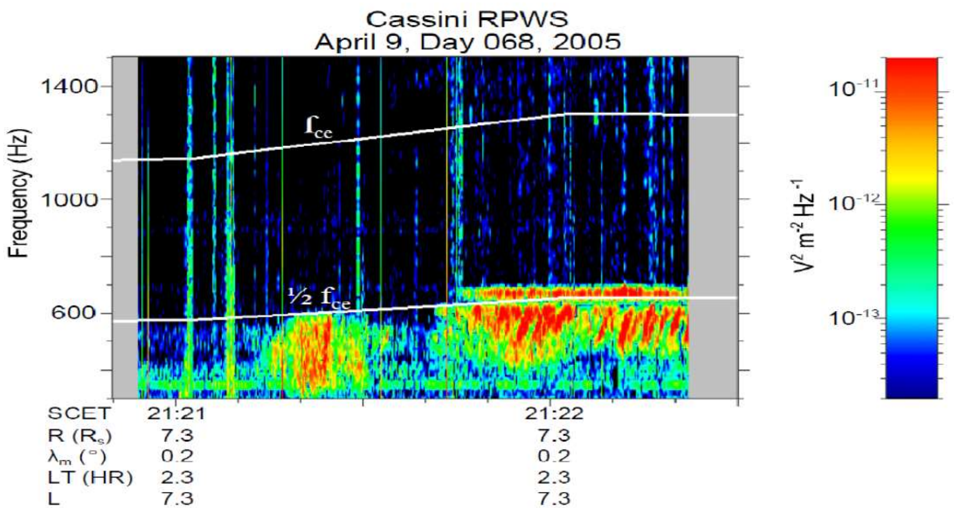


[*Omura et al., JGR, 2008*]

[*Hikishima et al., JGR, 2009*]

Observations:
oblique propagation





[*Santolik, et al., JGR, 2003*]

[*Hosphodarsky et al., JGR, 2008*]

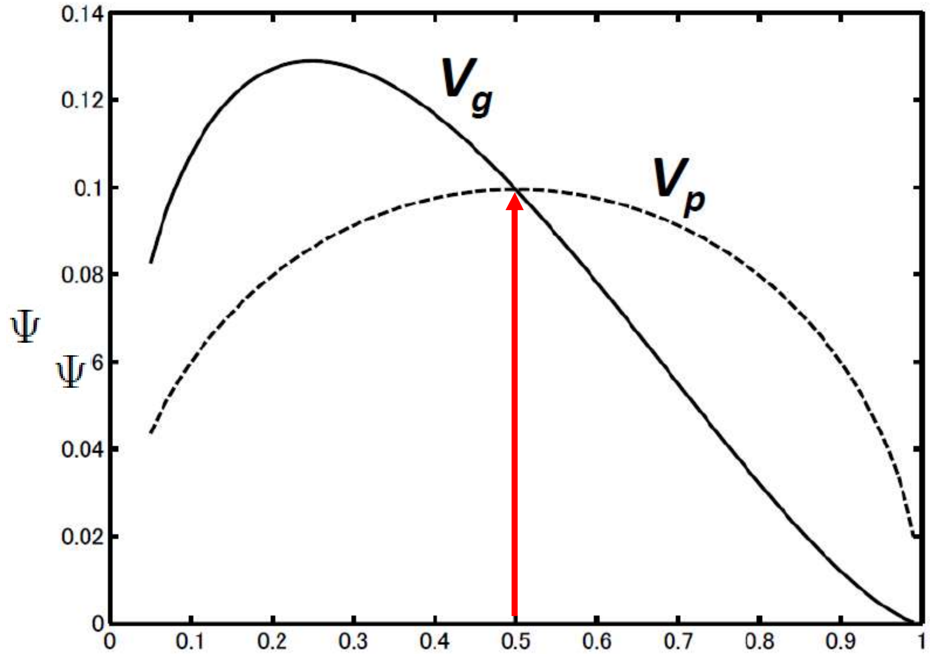
Quasi-parallel Propagation (Oblique) $\sin^2\Psi << 1$

Ψ: Wavenormal Angle

$$E_{w\parallel} = \frac{\omega \sin \Psi}{\delta^2 \Omega_e - \omega} E_w$$

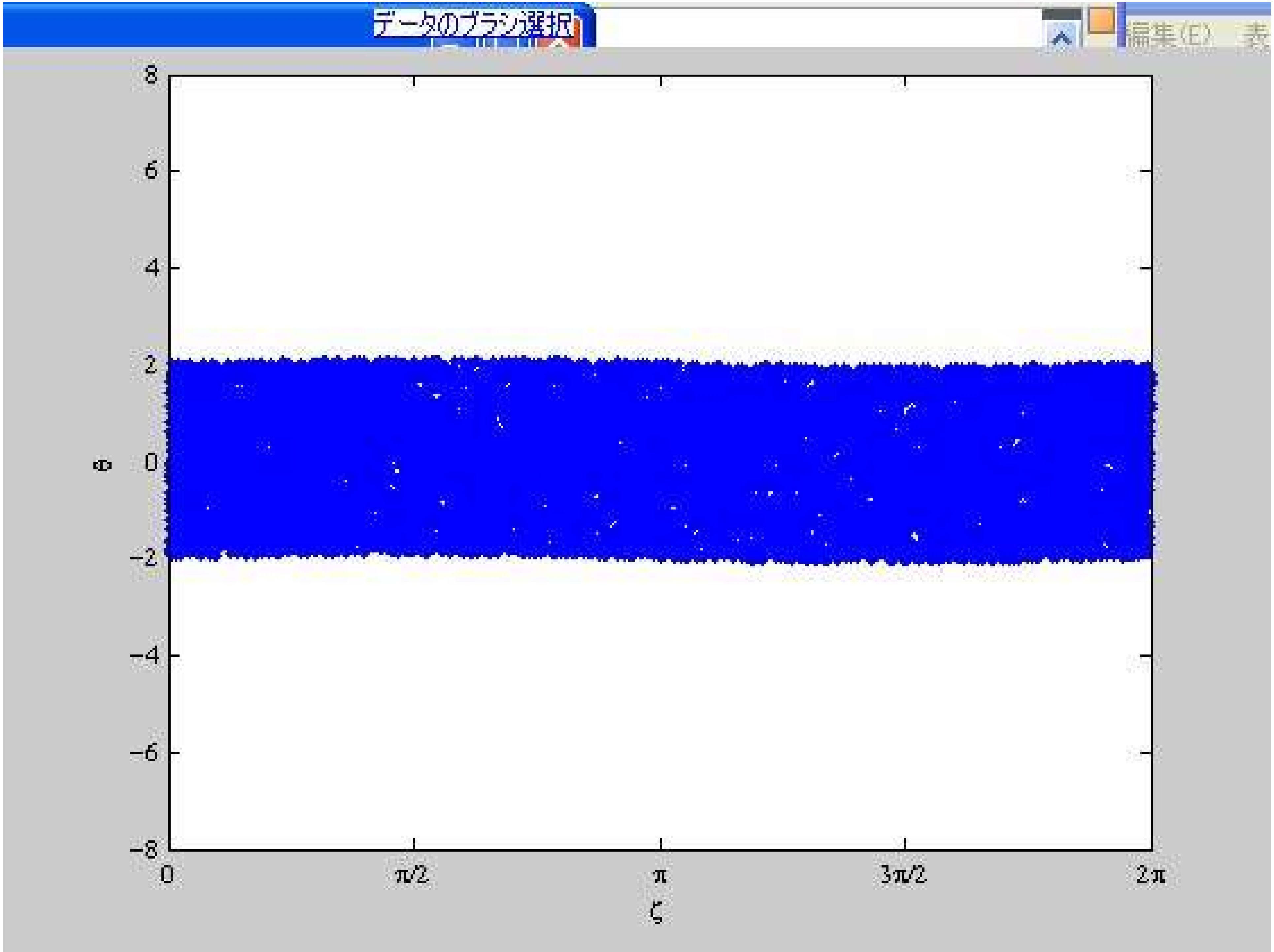
With $\omega = 0.5\Omega_e$

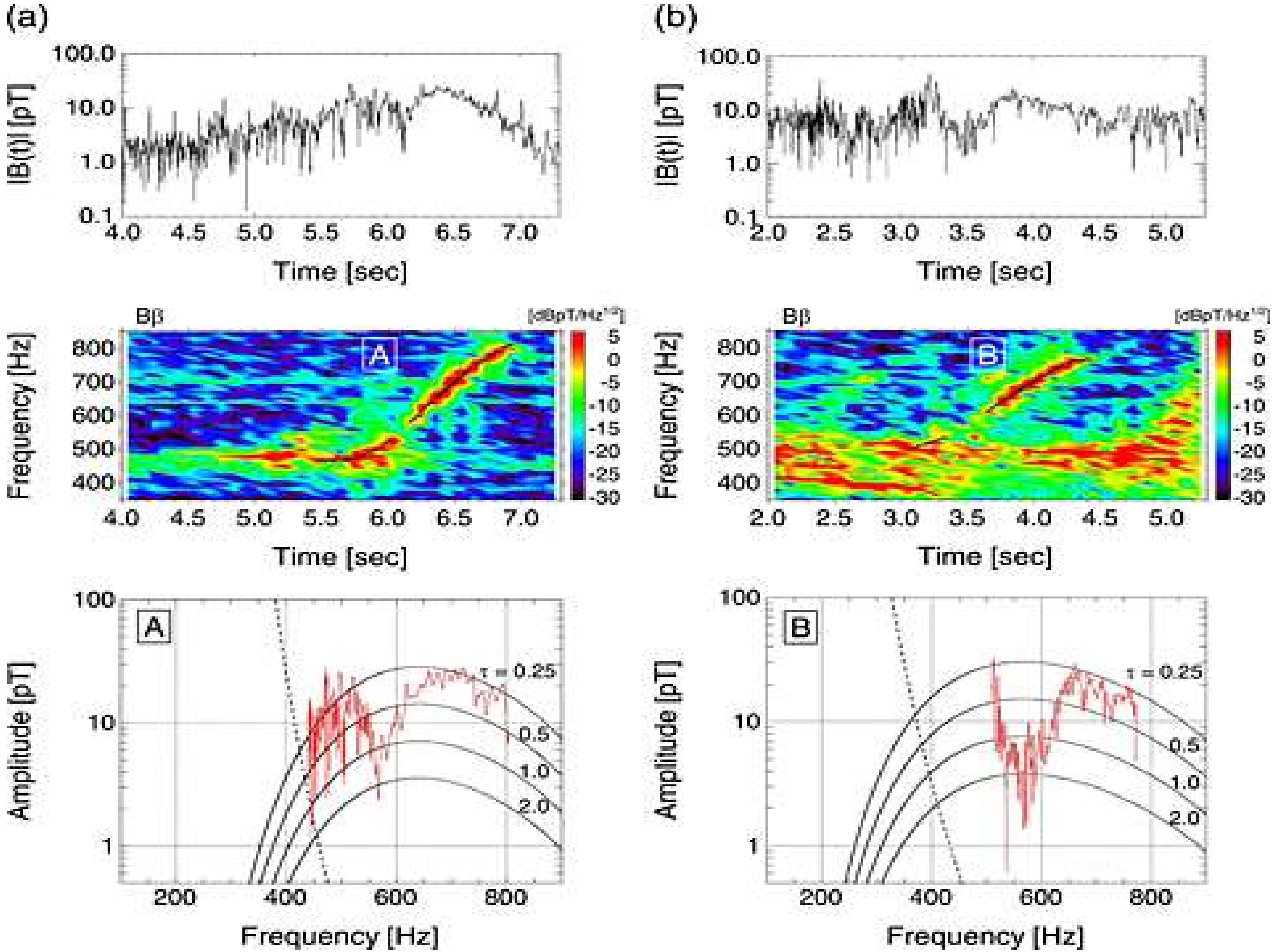
$$\boxed{V_g = V_p}$$



$$\tilde{v}_\parallel = v_\parallel - V_p$$

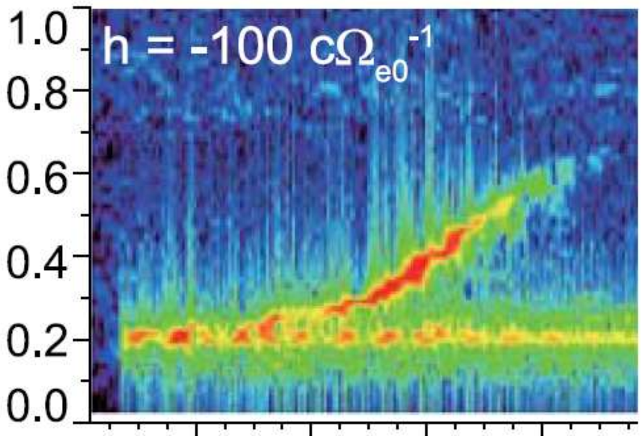
$$\frac{d\tilde{v}_\parallel}{dt} = -\frac{eE_{w\parallel}}{\gamma m_0} \sin\phi - \frac{v_\perp^2}{2\Omega_e}\frac{\partial \Omega_e}{\partial h}$$

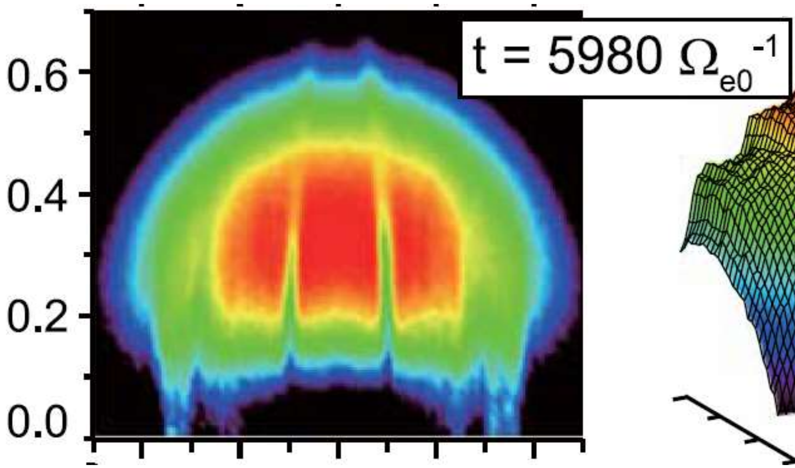

データのブラシ選択
編集(E)
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ

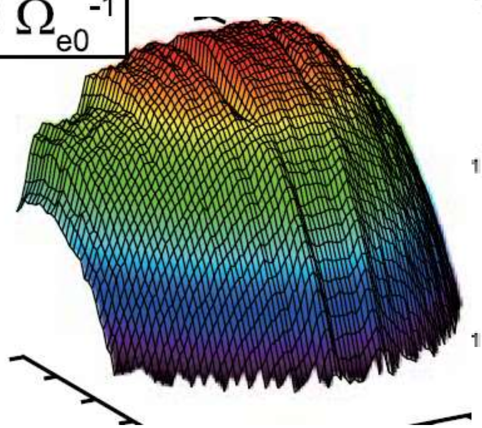


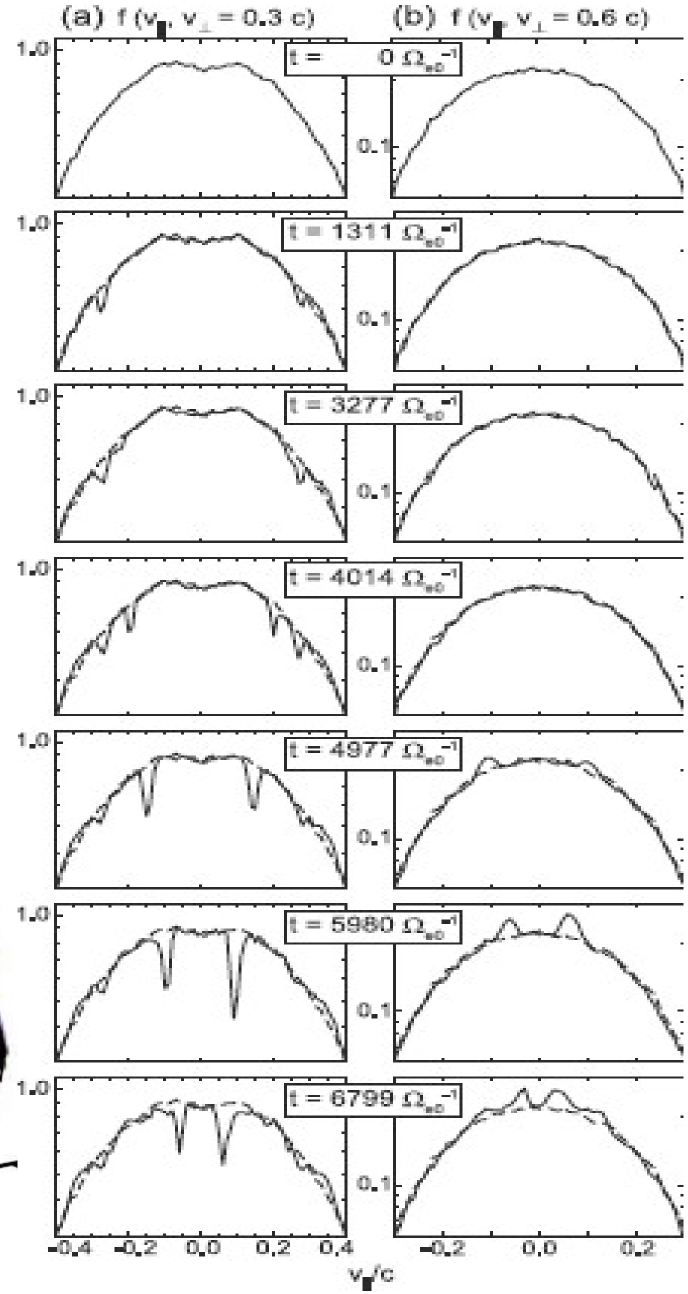
[*Yagitani et al., JGR, 2014*]

Formation of Electron Hole and Bump



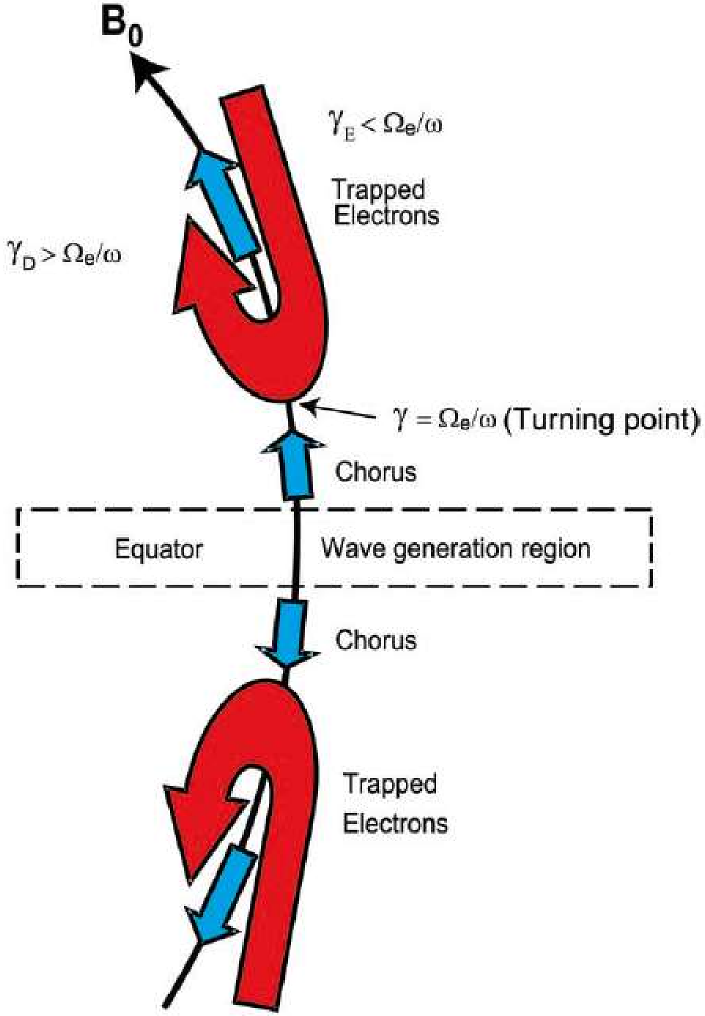




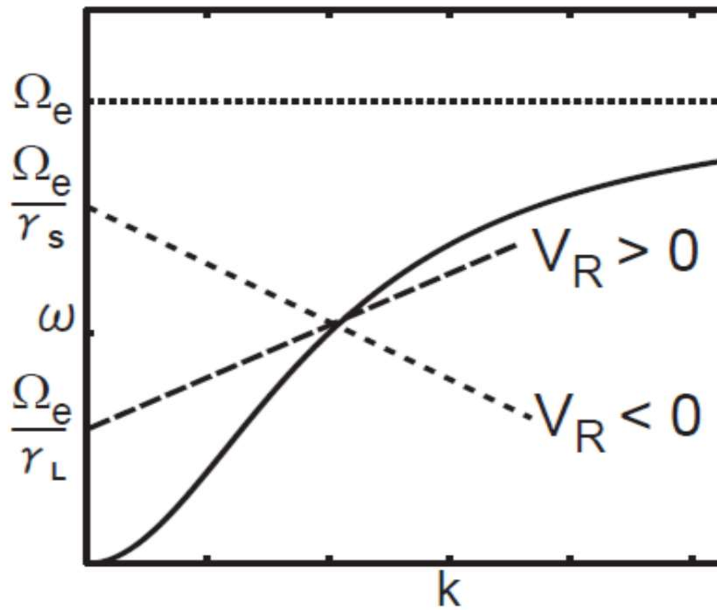


[*Hikishima et al., JGR, 2010*]

Relativistic Turning Acceleration



$$V_R = \frac{\omega}{k}\left(1 - \frac{\Omega_e(h)}{\omega\gamma}\right)$$

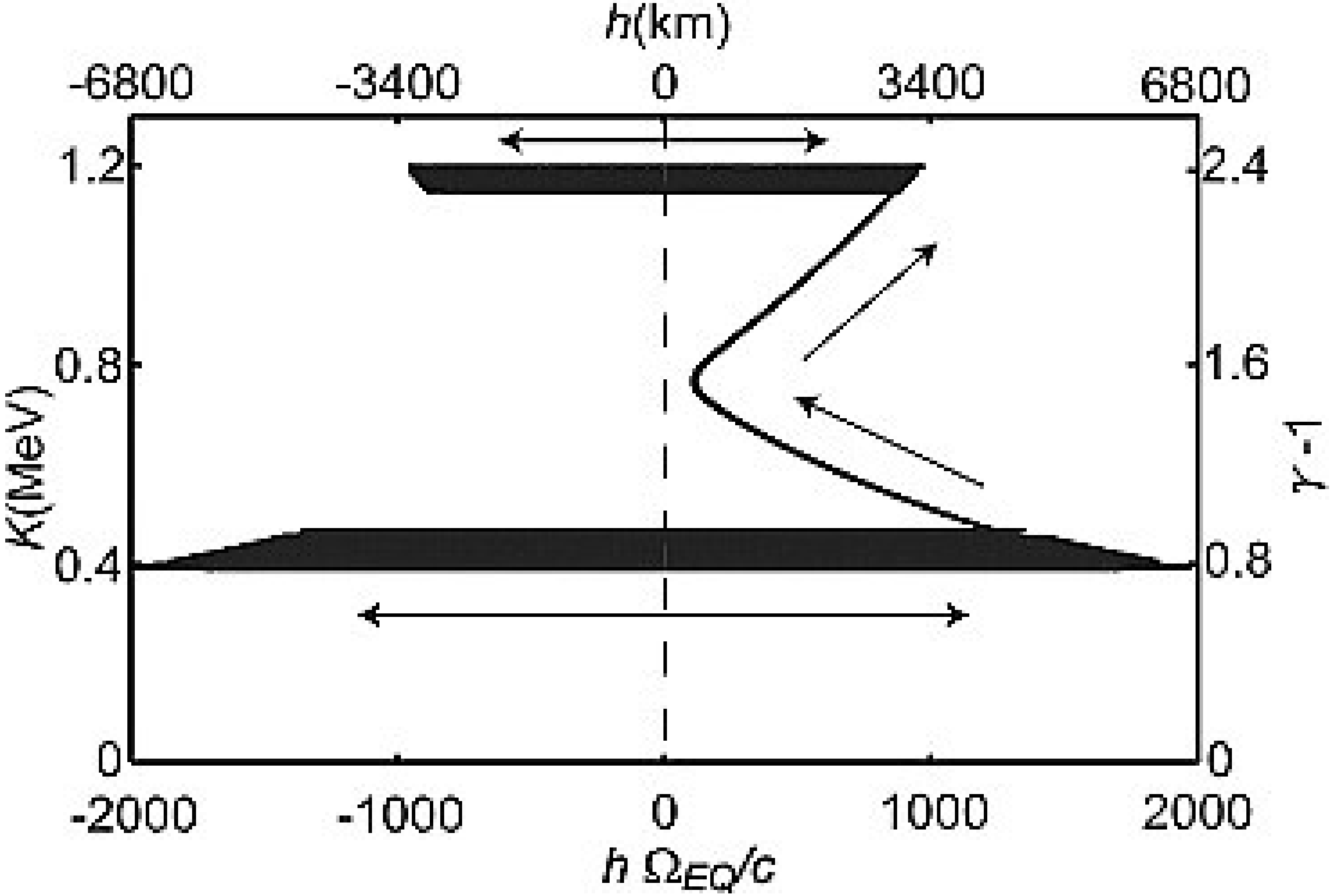


Ultra-Relativistic Acceleration $\gamma \geqq \Omega_e(h)/\omega \Rightarrow V_R \geqq 0$

[Omura , Furuya, Summers, JGR, 2007]

Trajectories of Resonant Electrons (400 keV)

Relativistic Turning Acceleration （RTA)



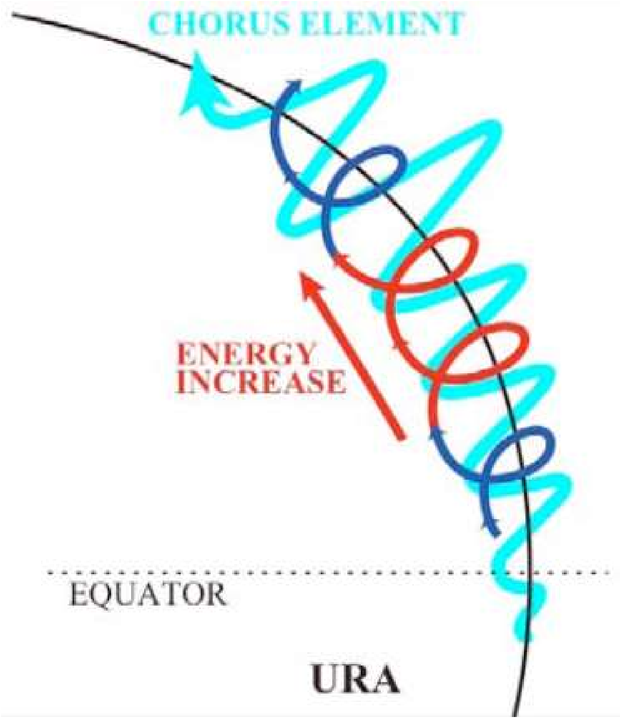
Bw = 125 pT $\omega_p = 2.0\Omega_{e0}$ $\omega = 0.4\Omega_{e0}$

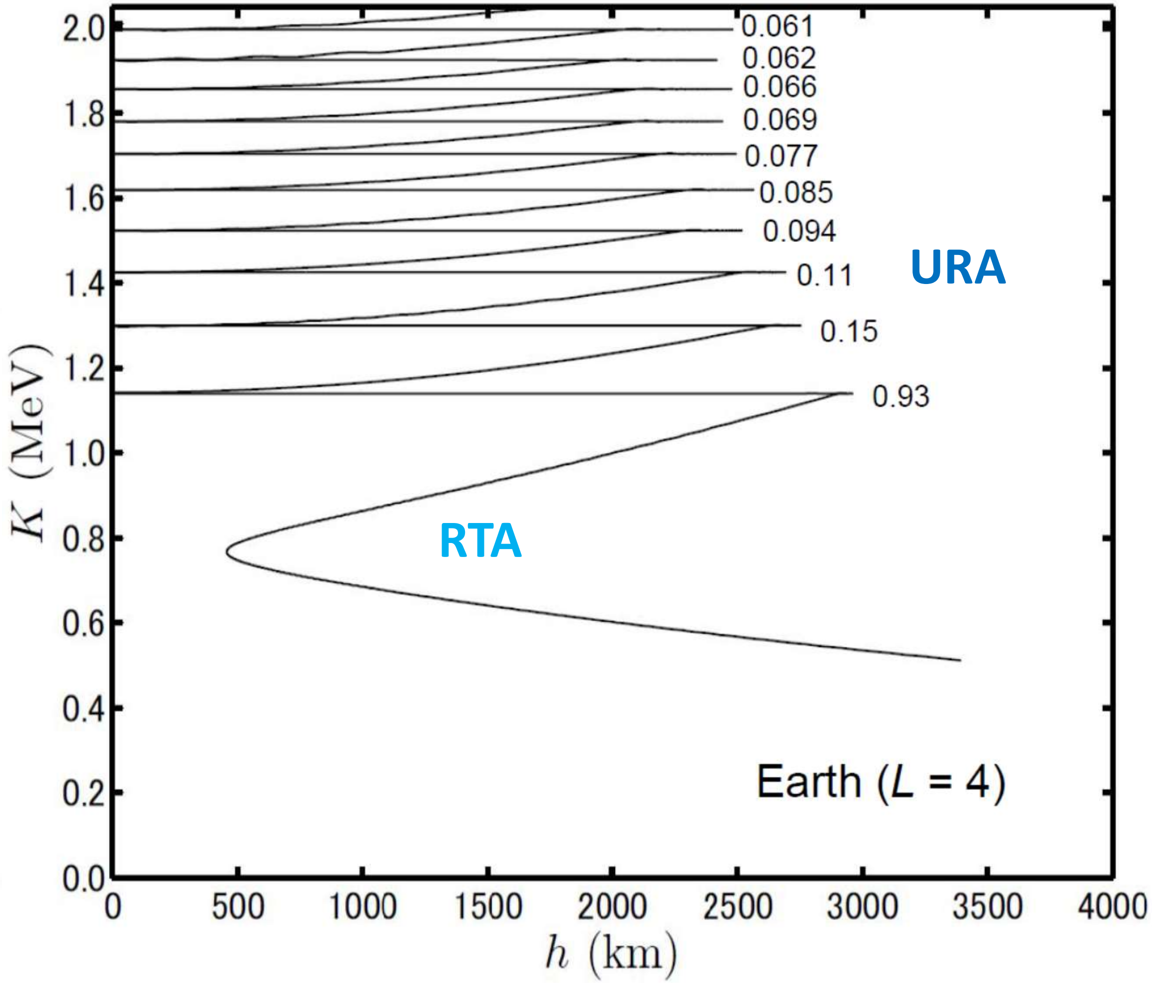
[*Omura, et al., JGR, 2007*]

Ultra-Relativistic Acceleration (URA)

$$\gamma_0 > \Omega_{EQ}/\omega$$

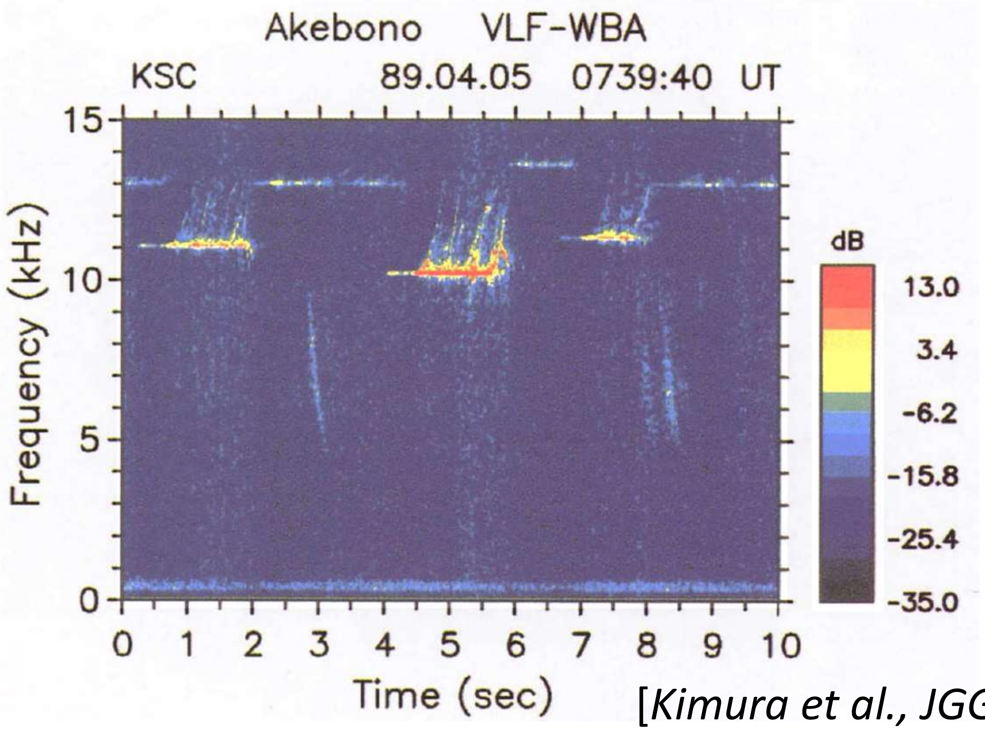
$$V_{R0} > 0$$





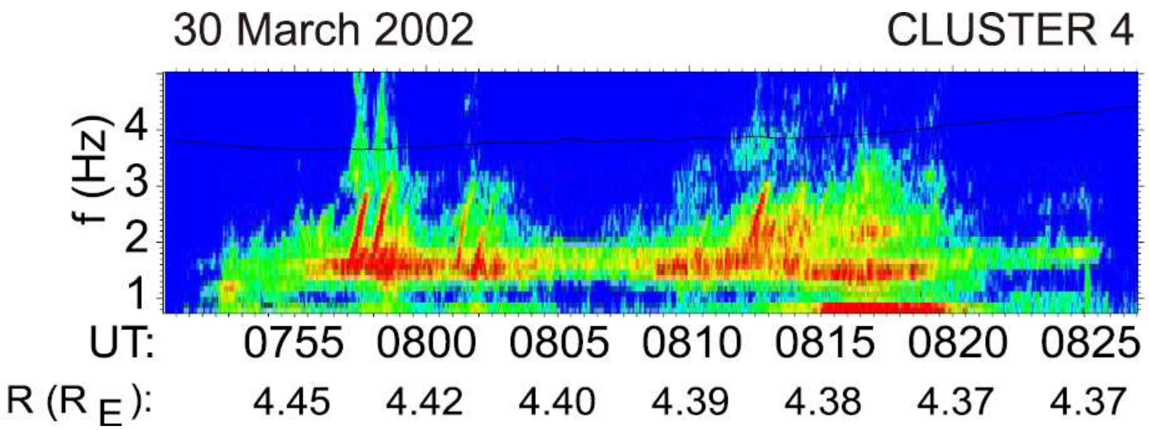
[Summers and Omura, GRL, 2007]

Whistler-mode Triggered Emissions

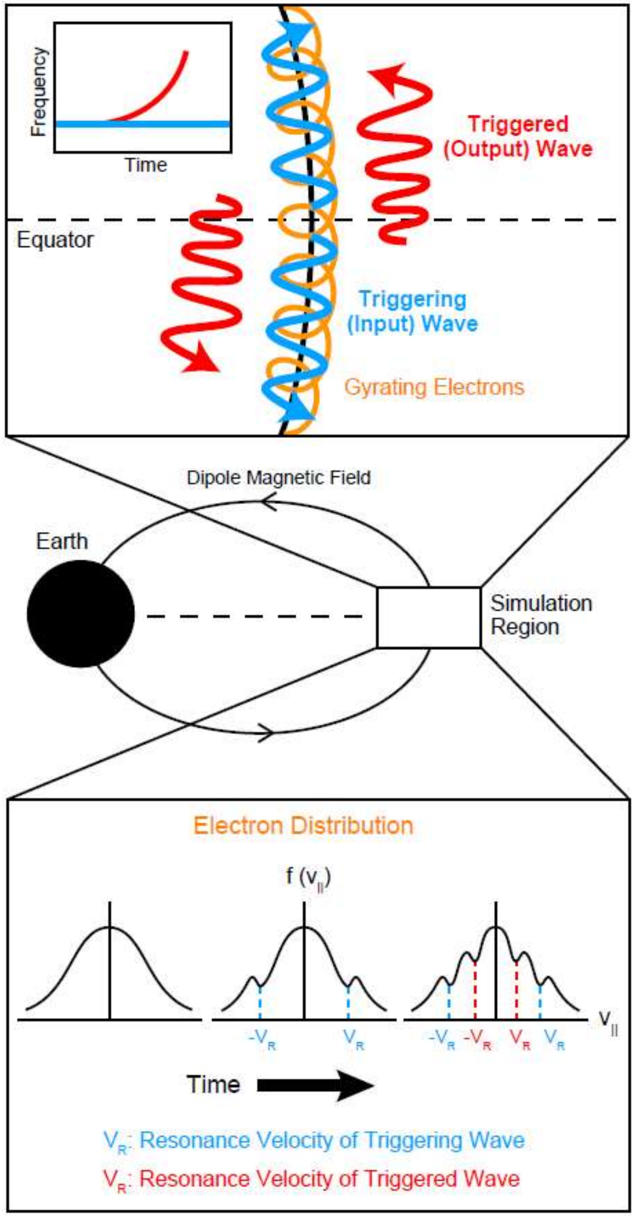


[*Kimura et al., JGG,* 1990]

EMIC Triggered Emissions

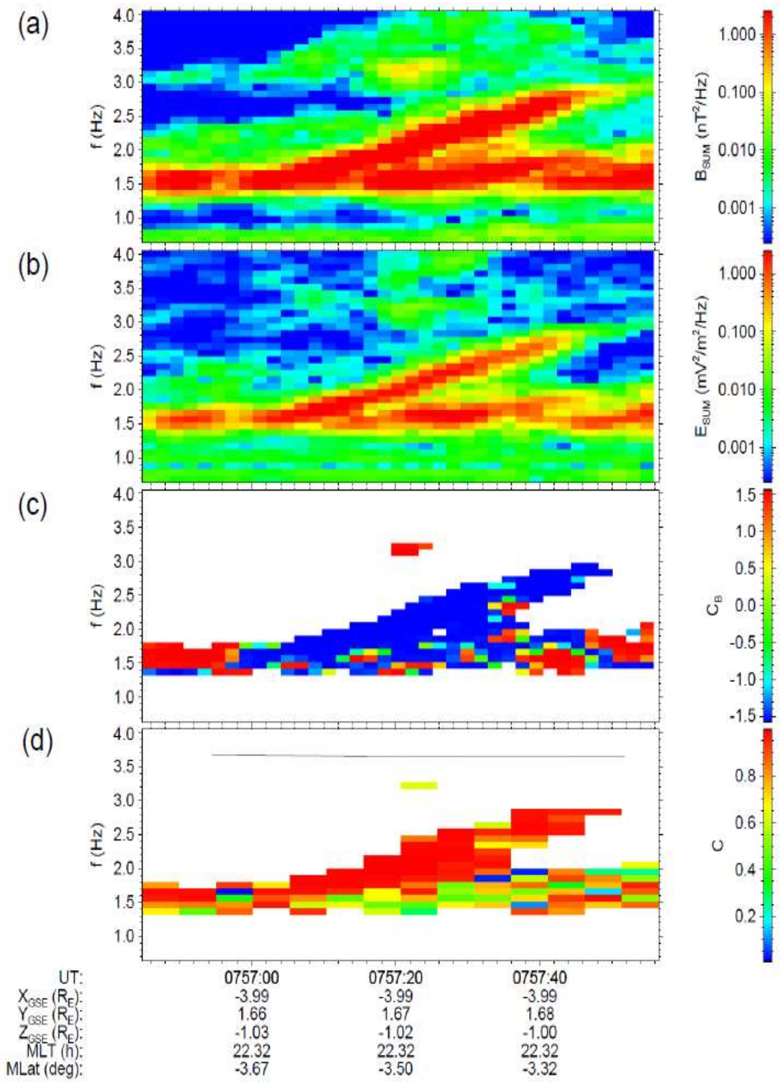


[*Pickett et al., GRL, 2010*]



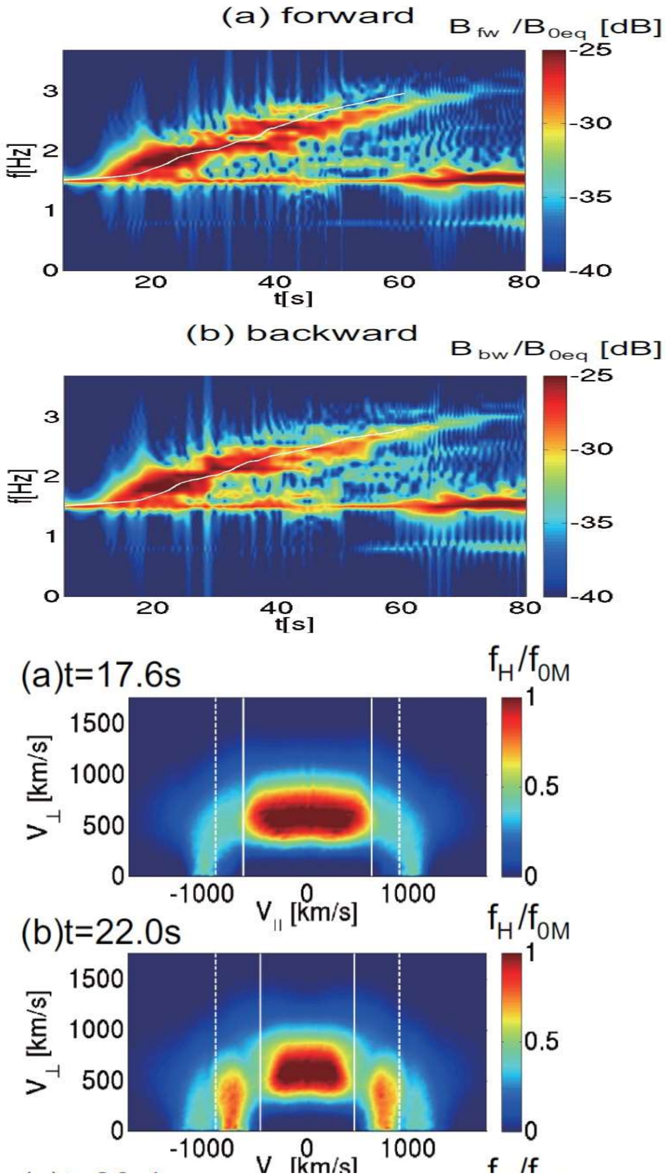
EMIC Triggered Emissions

Cluster Spacecraft Observation

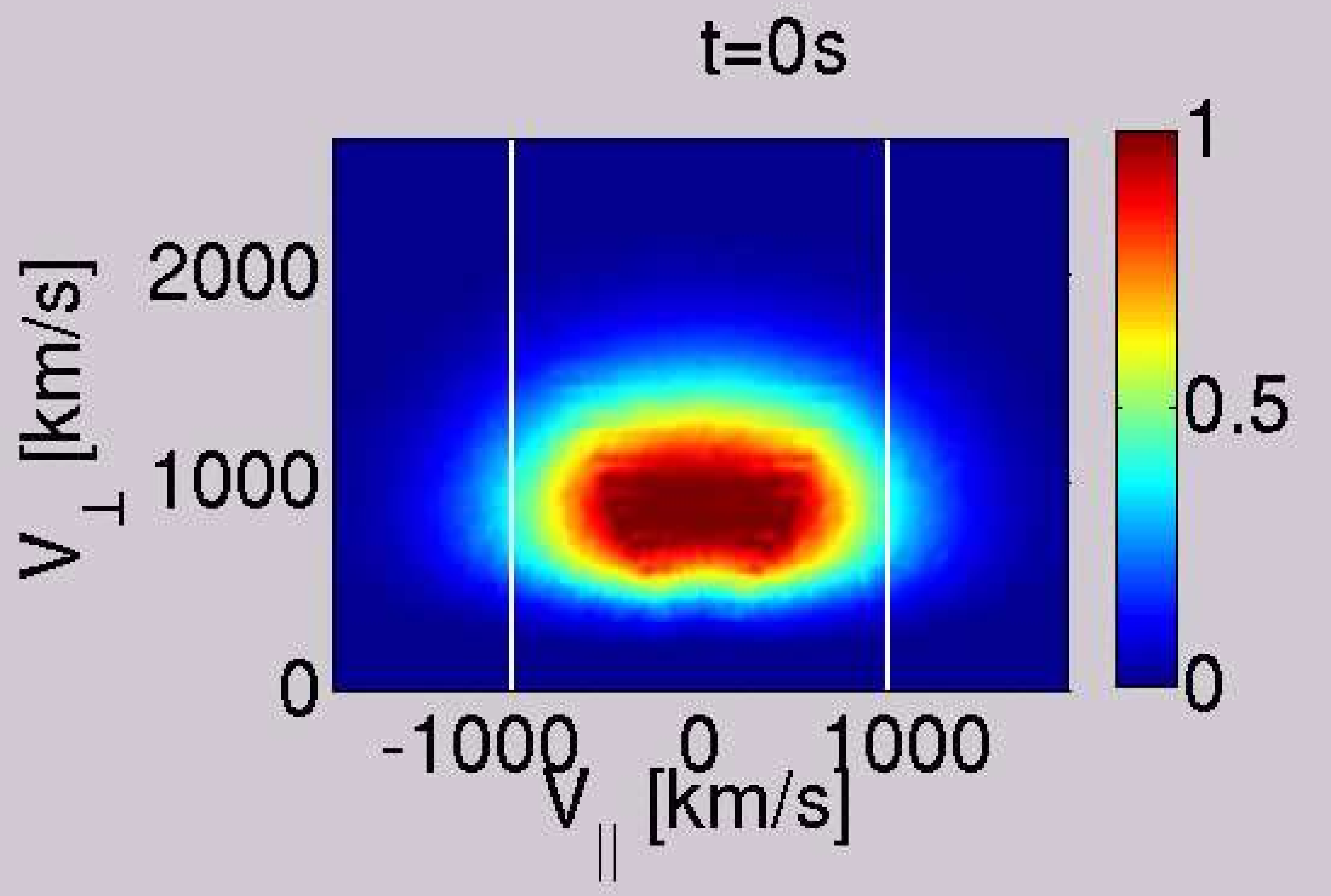


[*Omura et al., JGR, 2010*]

Hybrid Code Simulation

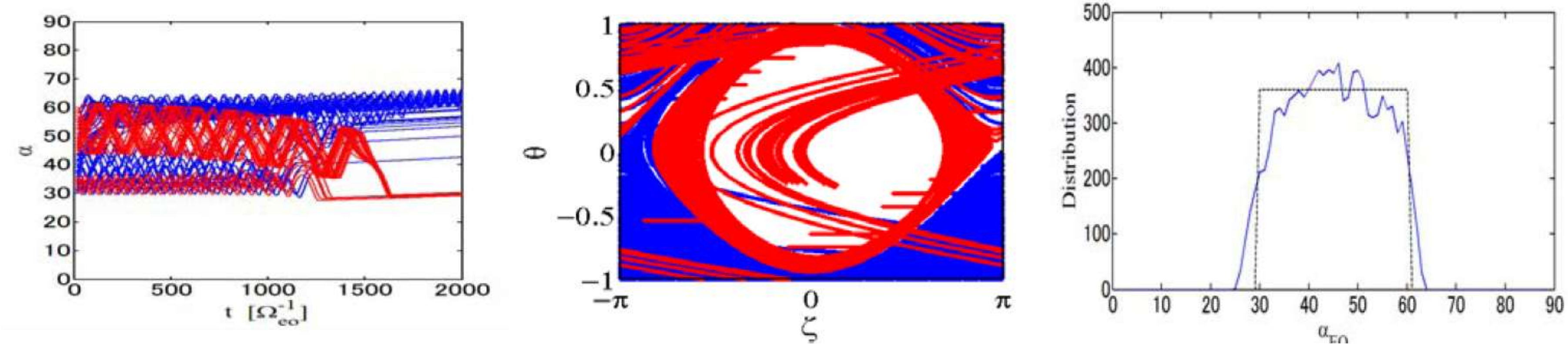


[*Shoji and Omura, JGR, 2011*]

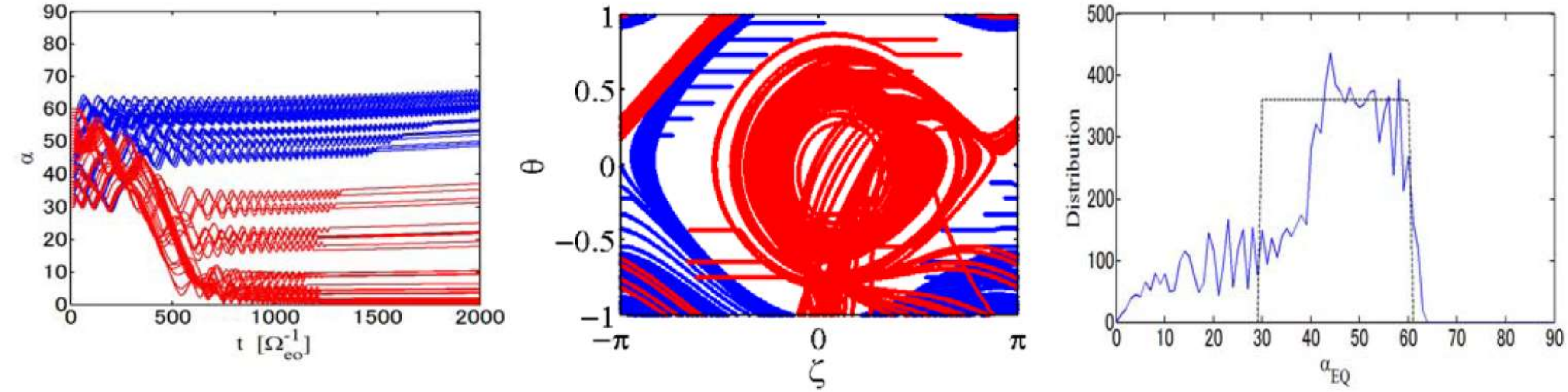

t=0s
V⊥ [km/s]
2000
1000
0
-1000
0
1000
V|| [km/s]
1
0.5
0

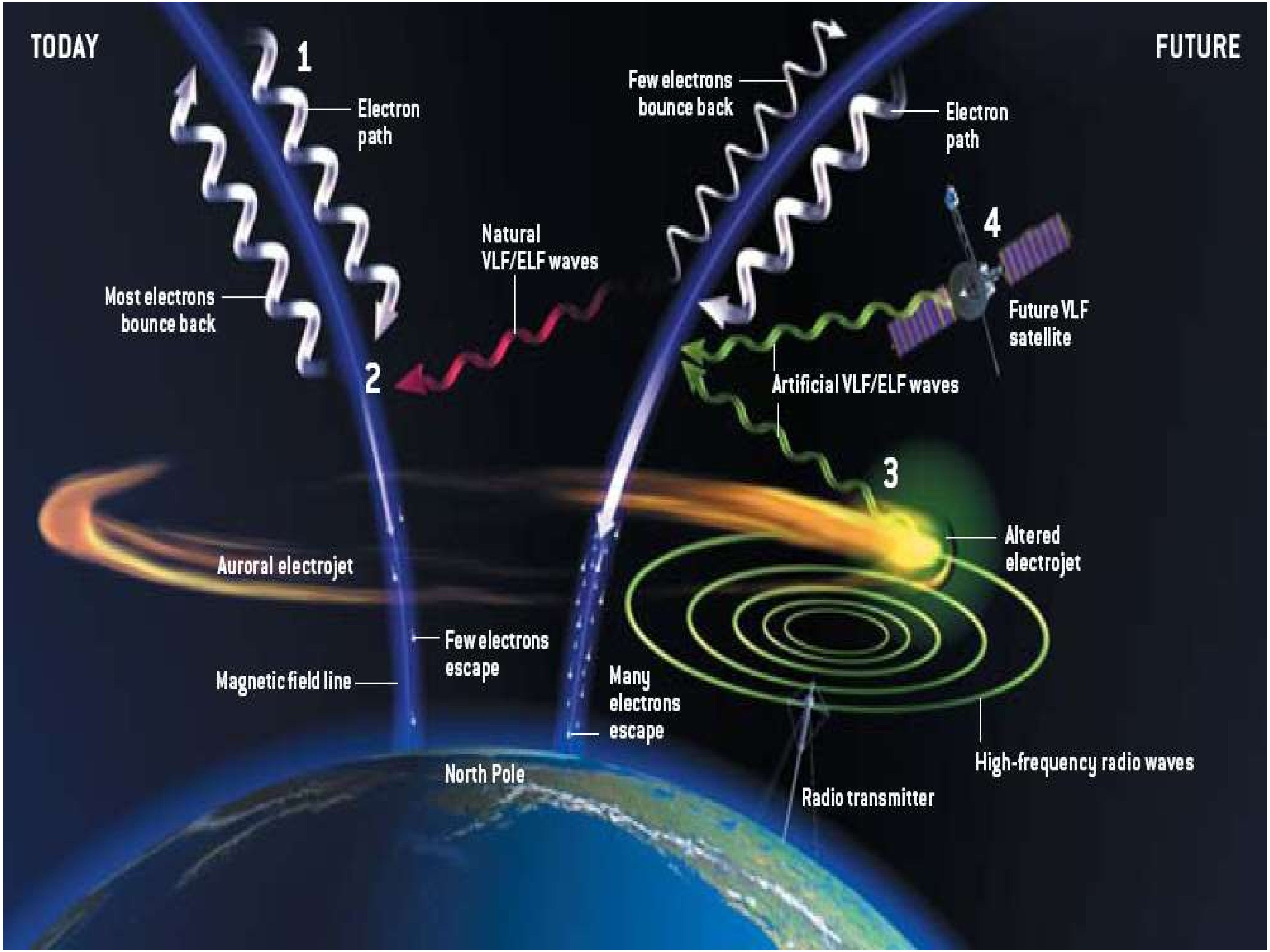
Test Particle Simulation of EMIC Wave-Electron Interaction

(a) Case A (0.98MeV, 2.8Hz-2.8Hz, a = 0.8e-7)

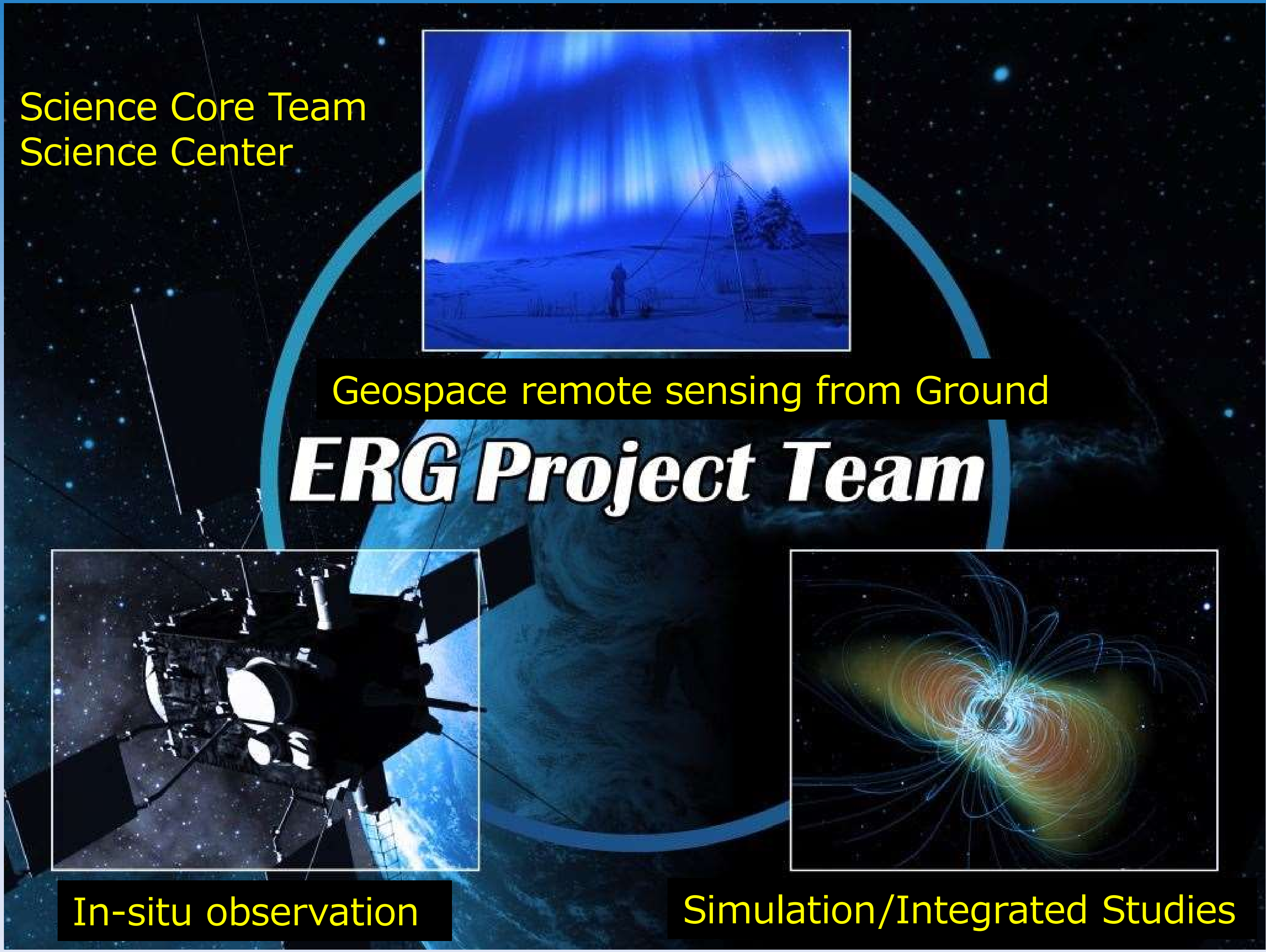


(d) Case D (0.98MeV, 2.8Hz-1.7Hz, a = 0.8e-7)




TODAY
FUTURE
1
Electron path
Most electrons bounce back
2
Natural VLF/ELF waves
Few electrons bounce back
Electron path
4
Future VLF satellite
Artificial VLF/ELF waves
3
Altered electrojet
Auroral electrojet
Few electrons escape
Magnetic field line
Many electrons escape
North Pole
Radio transmitter
High-frequency radio waves

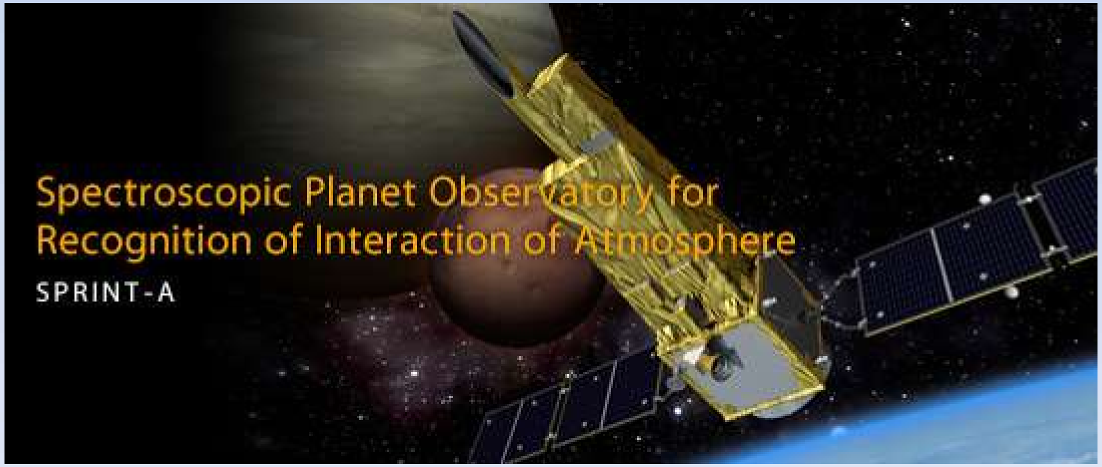
2. ERG Project Group



First launch of the Epsilon launch vehicle on September 13, 2013.

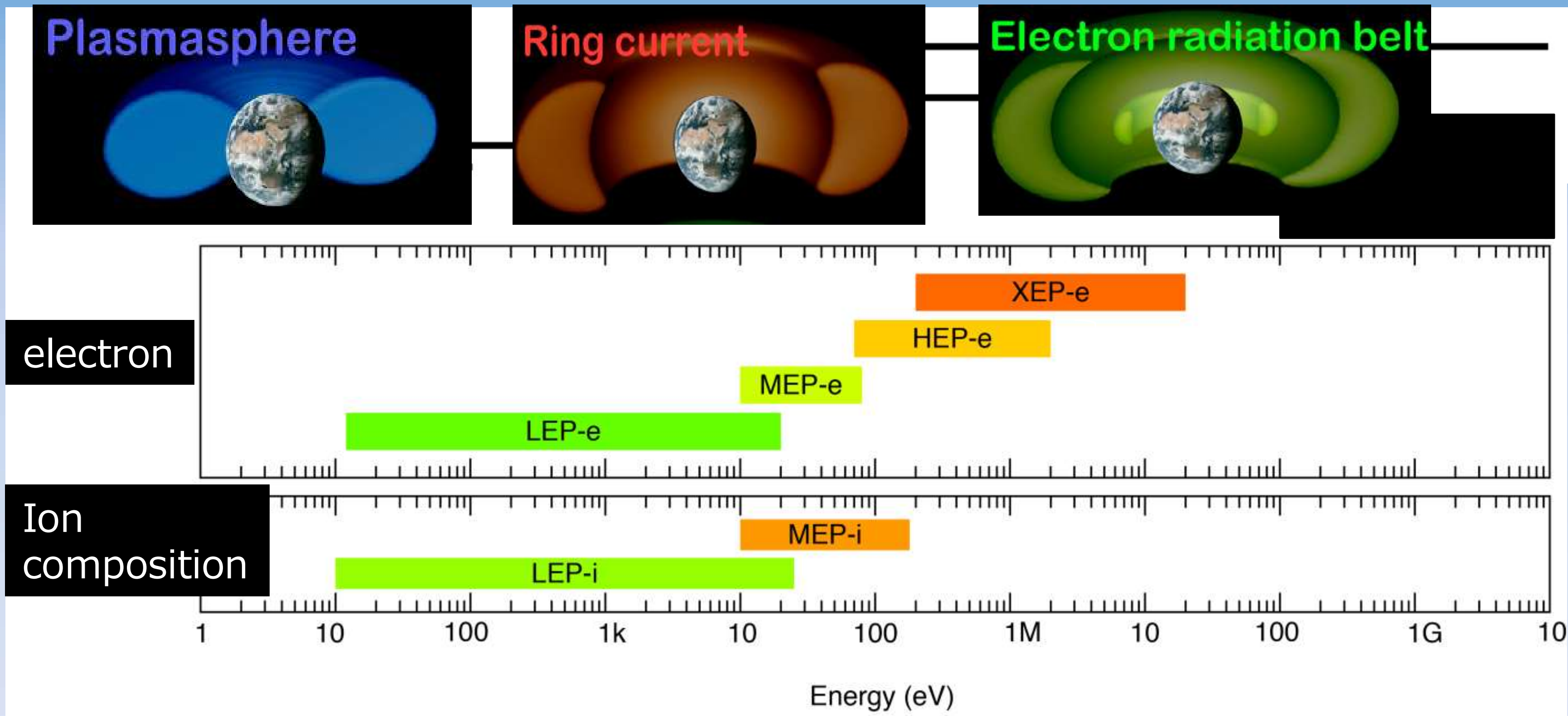
- Hisaki [Spectroscopic Planet Observatory] was successfully launched Epsilon that is newly developed solid rocket on Sep. 13, 2013.

- ERG is the second M-class mission to be launched by Epsilon.



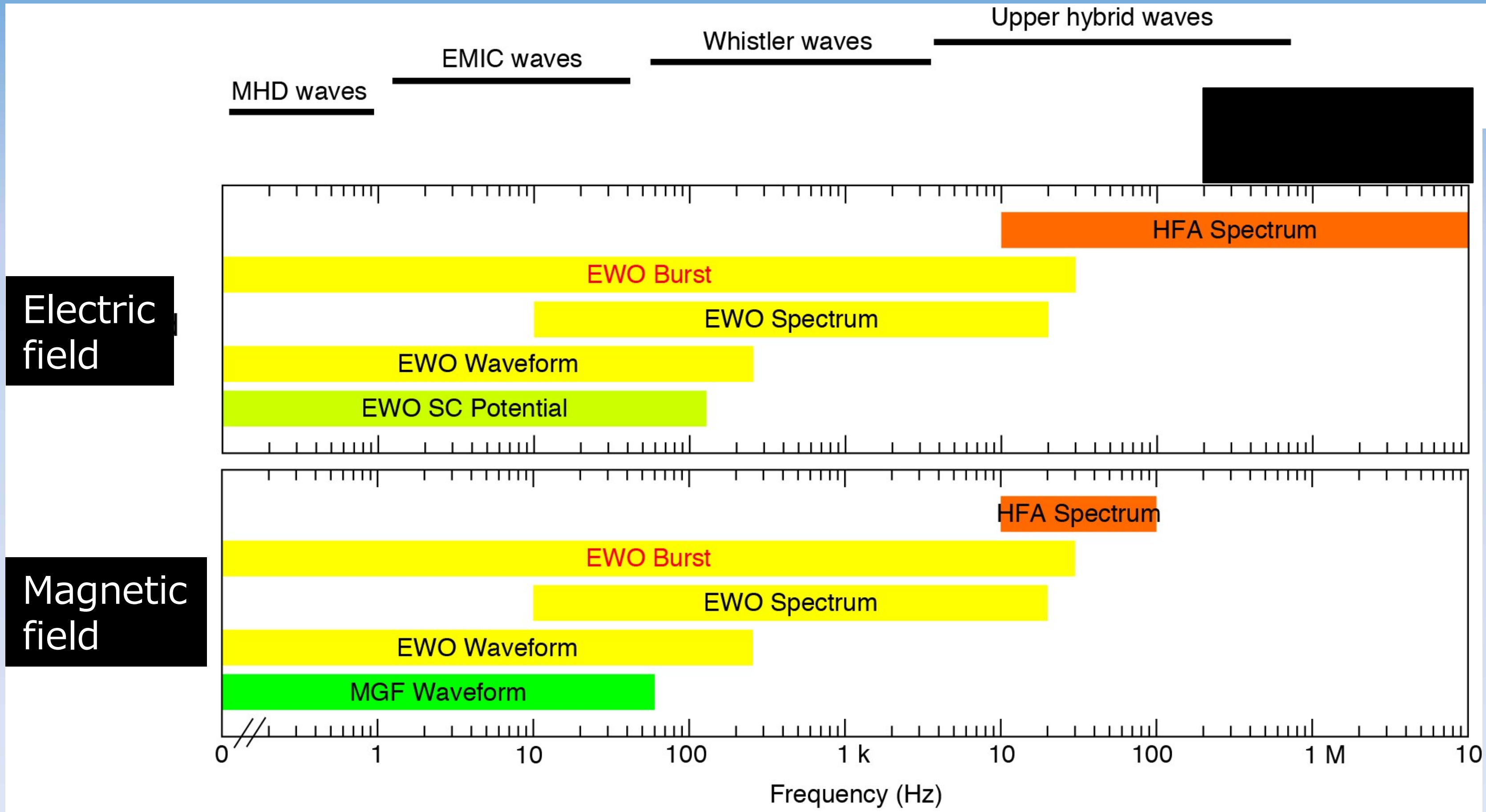
ERG : plasma & particles

PPE: Plasma and Particle Experiment Suite



ERG: Field and Waves

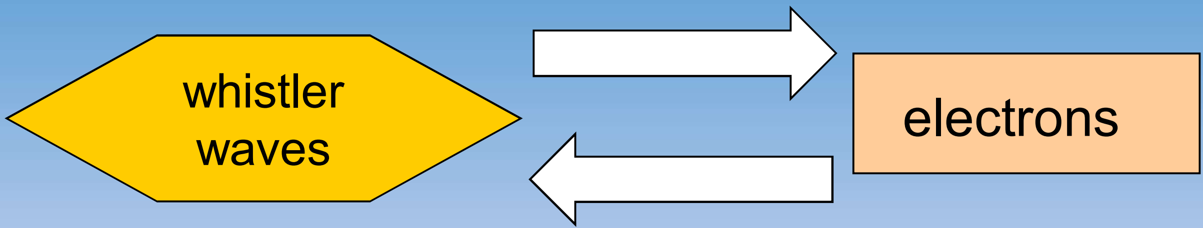
PWE: Plasma Wave and Electric Field Experiment
MGF: Measurement of Geomagnetic Field



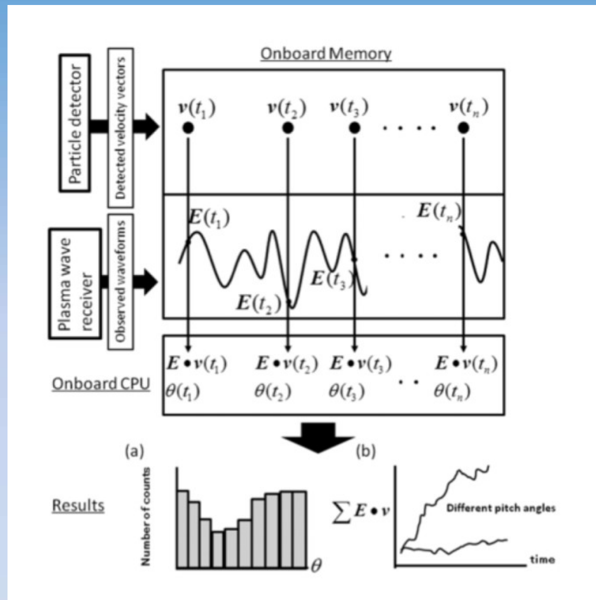
Wave-Particle Interaction Analyzer

S-WPIA: Software-type Wave Particle Interaction Analyzer

Direct measurement of energy transfer between whistler waves and electrons is essential to understand wave-particle interactions.



$$\frac{dK}{dt} = qE \cdot v = |E||v| \sin\theta$$



Phase difference θ determines the direction of energy transfer.
(Electrons generate waves or Waves accelerate electrons)

ERG-satellite will directly measure the energy transfer between whistler waves and electrons in space for the first time.

Mission Status & Schedule

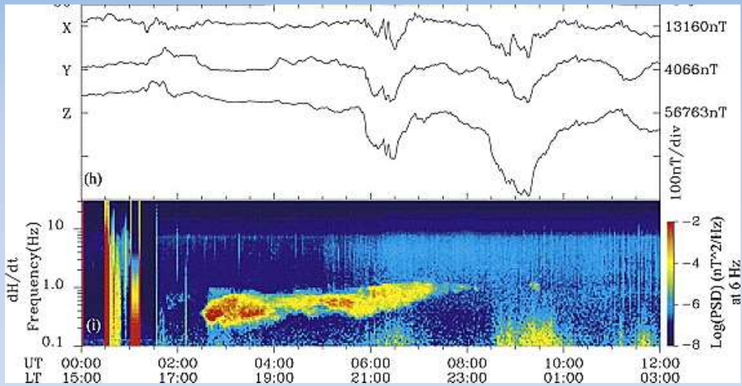
FY 2009	-	Mission Definition Review. System Requirement Review.
FY 2011	-	System Definition Review
FY 2012	-	Preliminary Design Review
FY 2013	-	Critical Design Review
FY 2014-15	-	Development of the flight model
2016	-	Launch of the satellite

The *ERG* ground networks : waves

・Radar Network: SuperDARN network

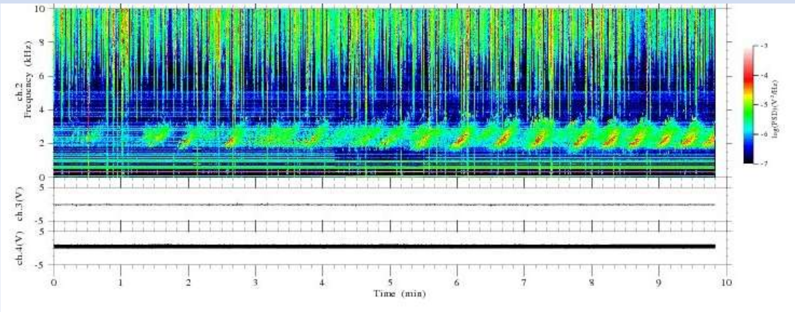
- global convective electric field
- ULF pulsation (Pc5)
- Electric field penetration

・Magnetometer Network : MAGDAS/CPMN,210MM Antarctica Network



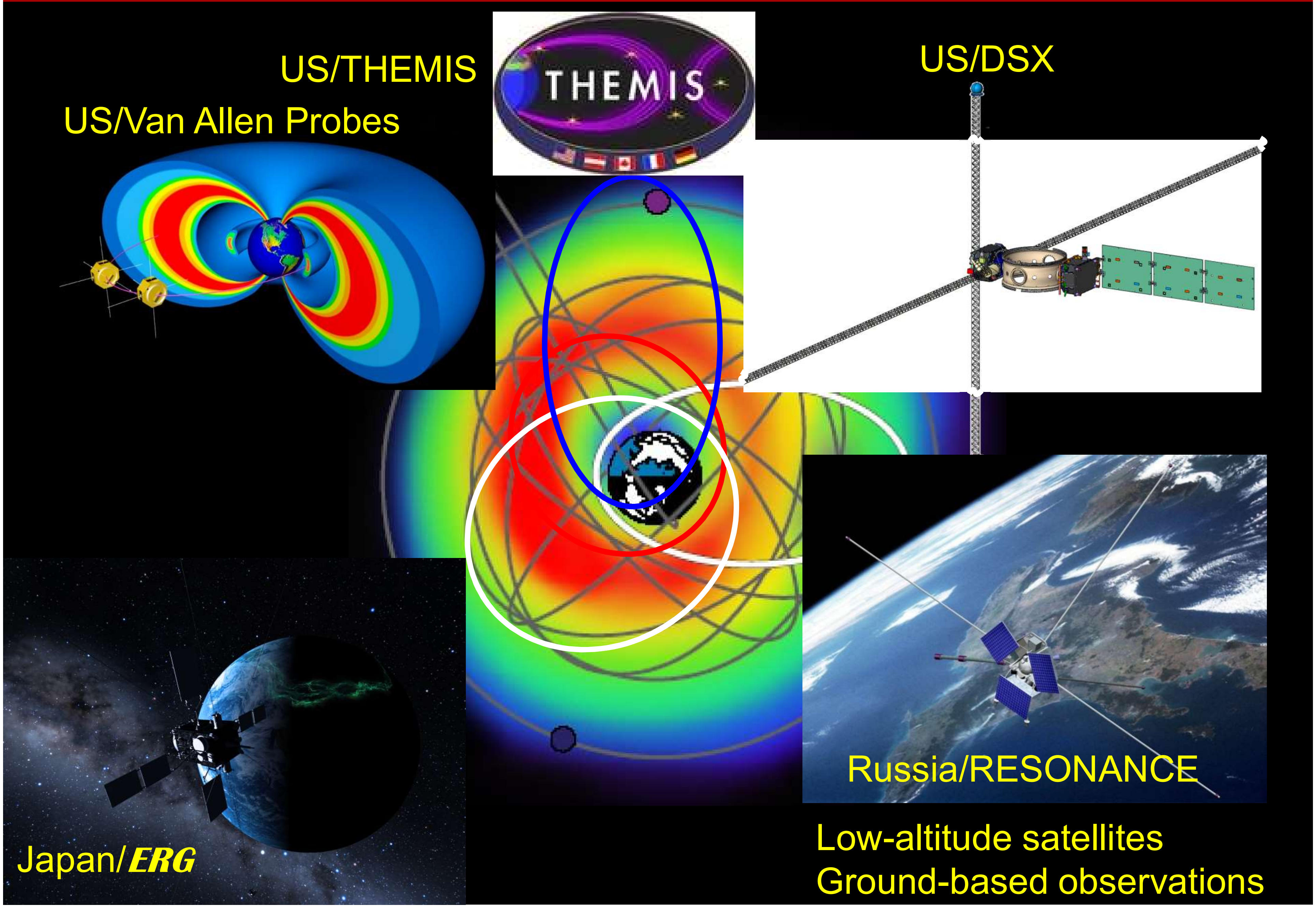
- ionospheric current /ring current.
- ULF pulsation (Pc5).
- EMIC (Pc1).
- diagnostics of plasmasphere

・VLF Network : Canada, Antarctica



- whistler (chorus, hiss).

3. International Collaboration: A golden era for geospace



レポート課題

1．磁気圏のシミュレーションは、宇宙天気における さまざまな擾乱を調べ、予報されることに利用されているが、今後より高い精度で計算可能となった場合、何か別の利用方法がないか考えよ。（地球以外の磁気圏でも可）

2．MHDシミュレーションと粒子シミュレーションの本質的な違いについて簡単に述べよ。