# 計算科学が拓く世界
# 地球・惑星・宇宙と計算科学2
# 宇宙プラズマ粒子シミュレーション

大村善治

京都大学生存圏研究所,

omura@rish.kyoto-u.ac.jp

# ブラゾフ方程式 → MHD方程式

速度分布関数 $f(\mathbf{x},\mathbf{v},t)$

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}} = \left.\frac{\partial f}{\partial t}\right|_{\mathrm{coll}}$$

$$n_\alpha = \int F_\alpha d\underline{v}$$

$$\underline{u}_\alpha = \frac{1}{n_\alpha}\int \underline{v} F_\alpha d\underline{v} = \langle \underline{v} \rangle$$

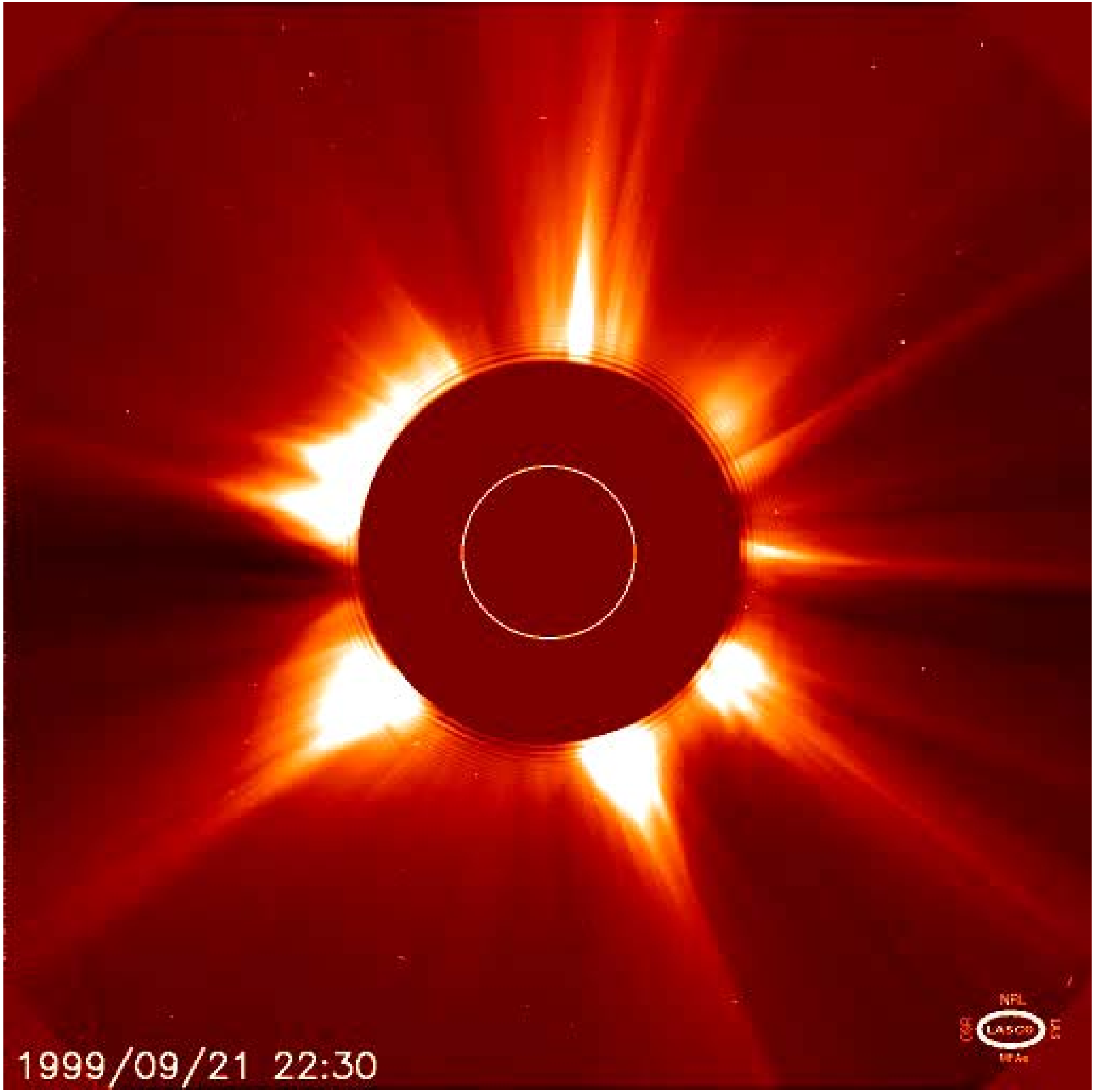
speed of light $\rightarrow \infty$

electron inertia $\rightarrow 0$

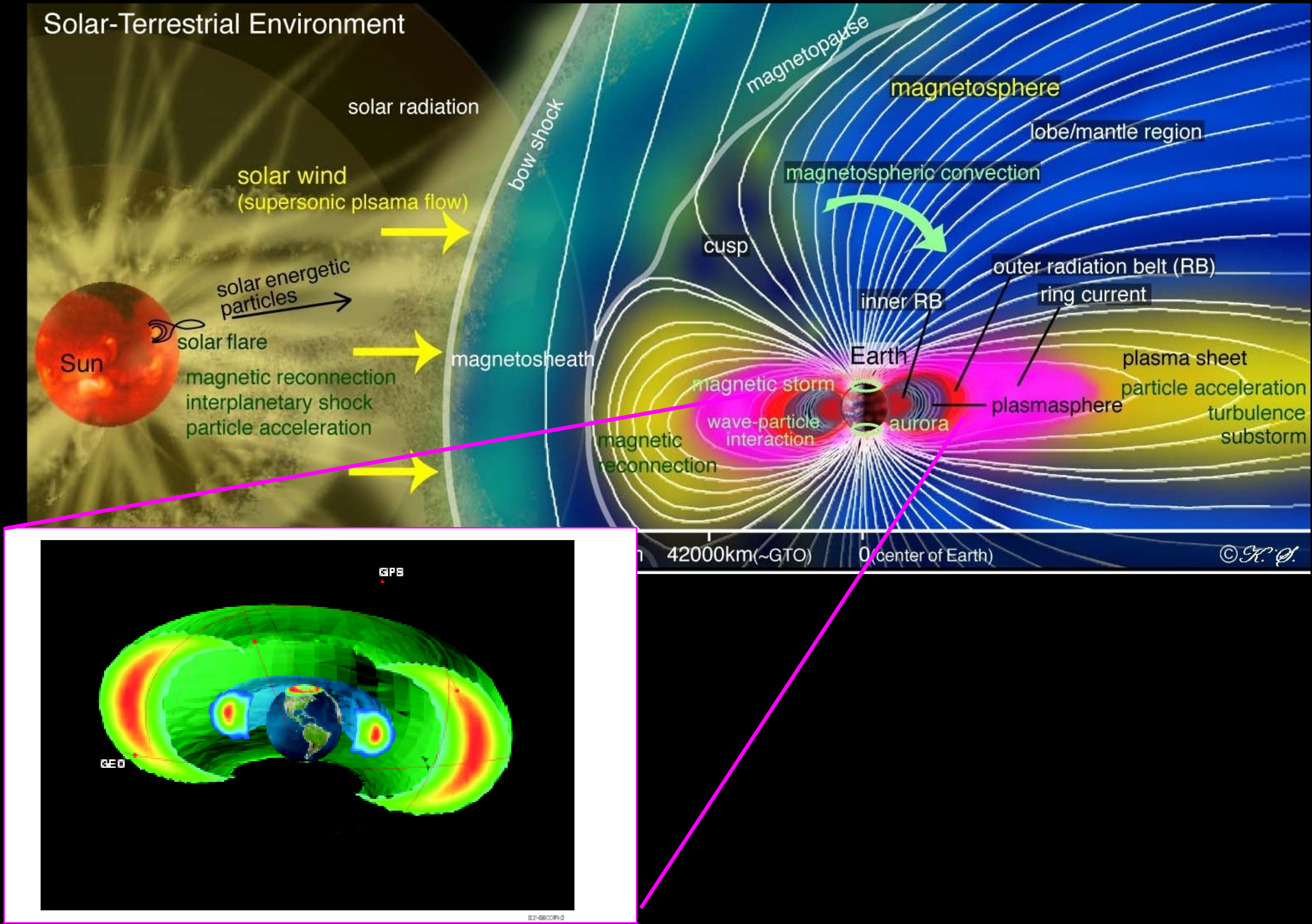
$$\overleftrightarrow{P}_\alpha = n_\alpha m_\alpha \langle \overleftrightarrow{\underline{v}}\,\overleftrightarrow{\underline{v}} \rangle$$

$$m_\alpha n_\alpha \frac{d\underline{u}_\alpha}{dt} = q_\alpha n_\alpha \left(\underline{E} + \underline{u}_\alpha \times \underline{B}\right) - \nabla\cdot \underline{\underline{P}}_\alpha + \underline{R}_\alpha$$

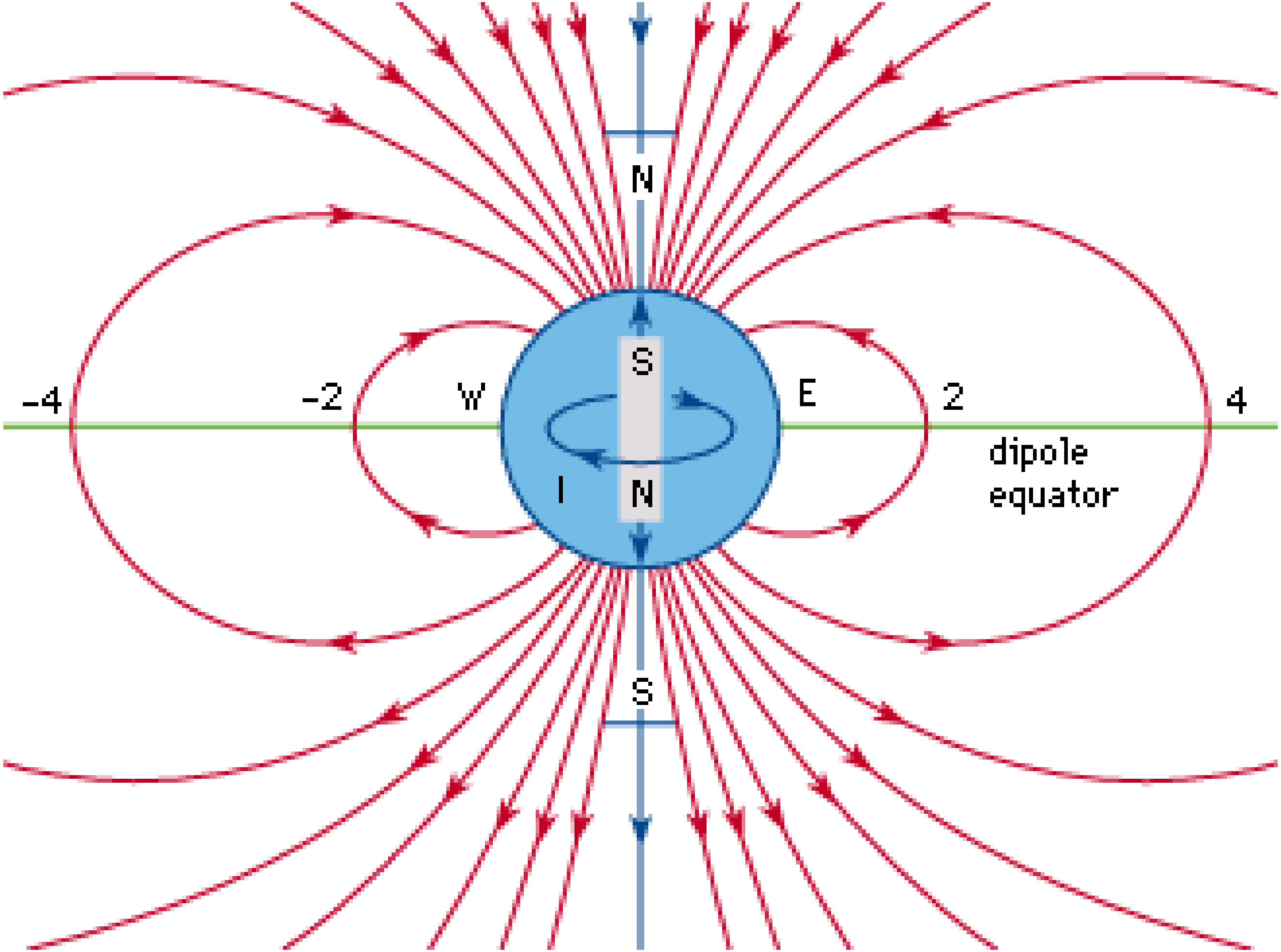
# 太陽風



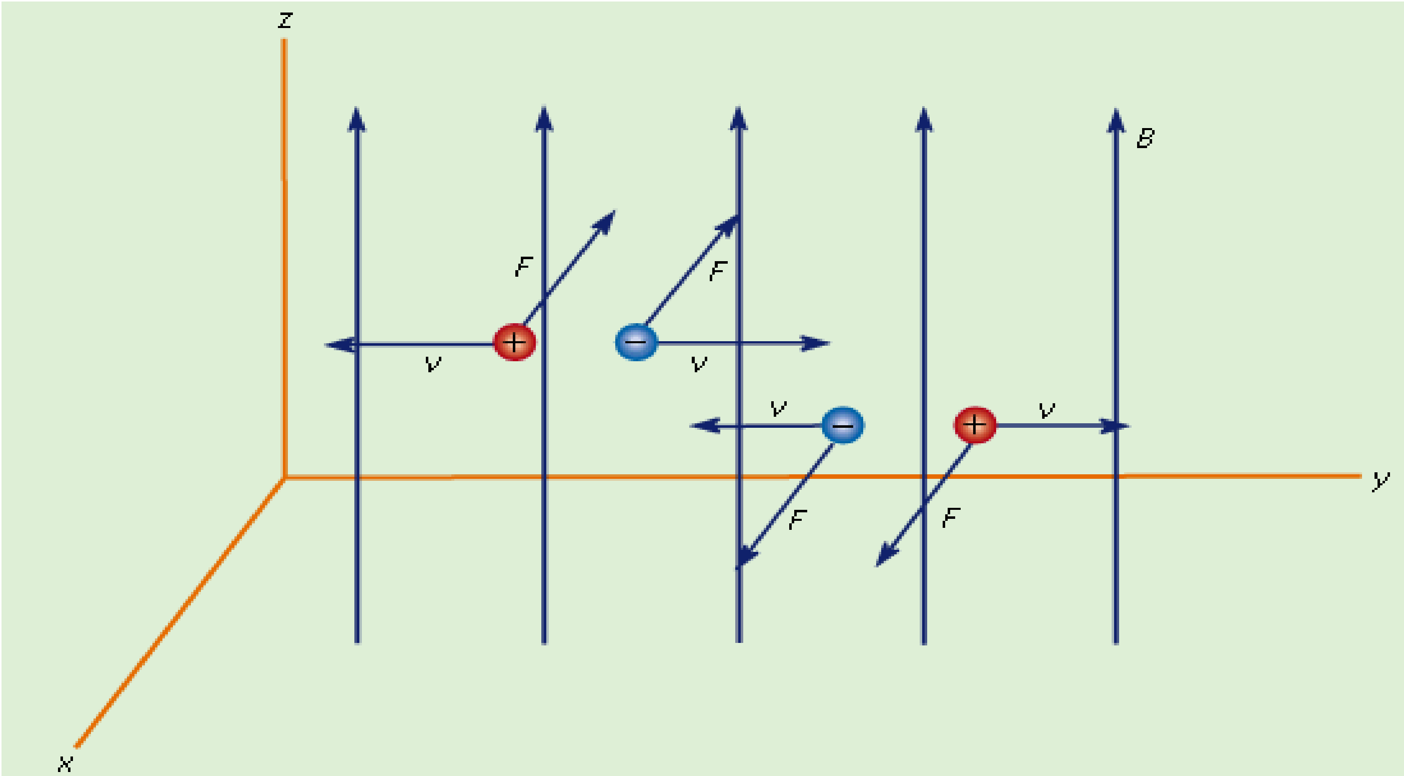
# Geospace



Solar-Terrestrial Environment
solar radiation
solar wind
(supersonic plsama flow)
solar energetic particles
solar flare
Sun
magnetic reconnection
interplanetary shock
particle acceleration
bow shock
magnetosheath
magnetopause
magnetosphere
lobe/mantle region
magnetospheric convection
cusp
outer radiation belt (RB)
ring current
inner RB
Earth
plasma sheet
particle acceleration
turbulence
substorm
magnetic storm
plasmasphere
wave-particle interaction
aurora
magnetic reconnection
42000km(~GTO)
0 (center of Earth)
GPS
GEO


N
S
N
S
-4
-2
W
E
2
4
I
dipole
equator

# Motion of Charged Particles （Lorentz Force）

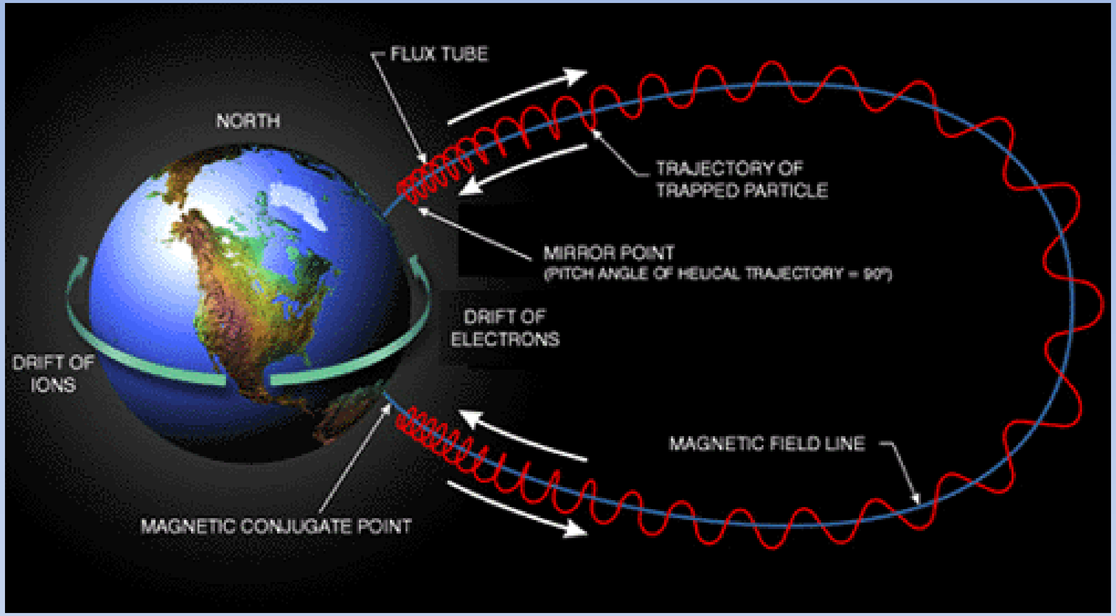


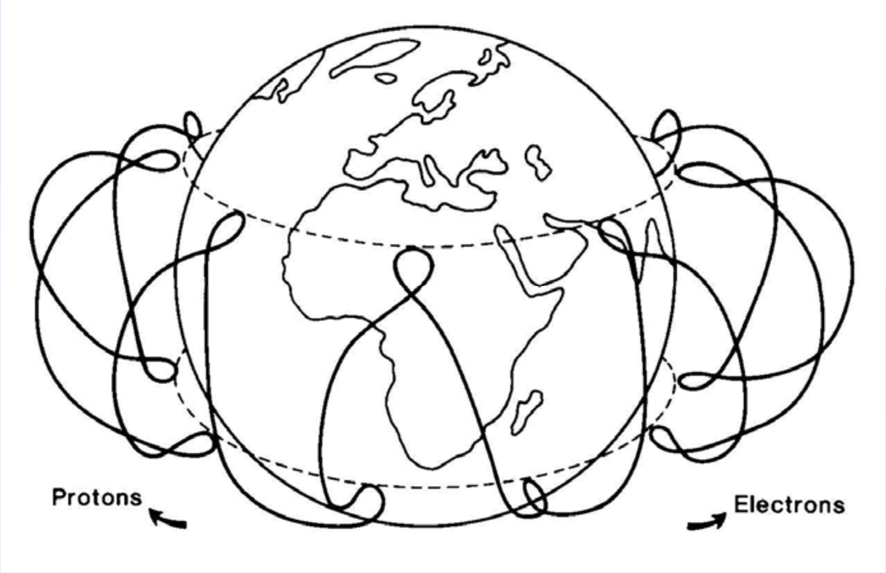

## Equation of Motion of Relativistic Electrons

$$m_0 \frac{\mathrm{d}(\gamma \boldsymbol{v})}{\mathrm{d}t} = -e[\boldsymbol{E}_w + \boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{B}_w)]$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

# Motion of Charged Particles in Dipole Magnetic Field



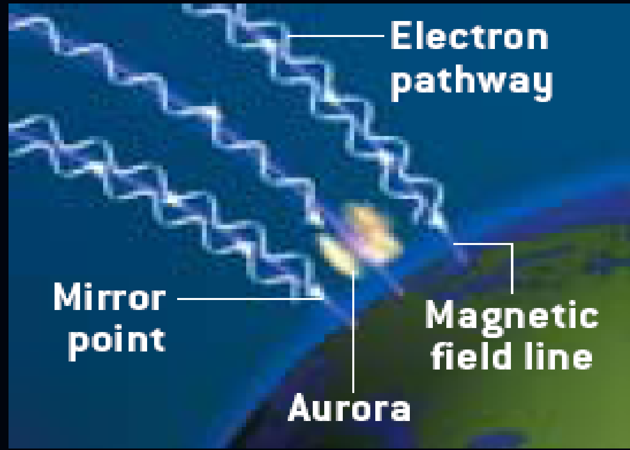



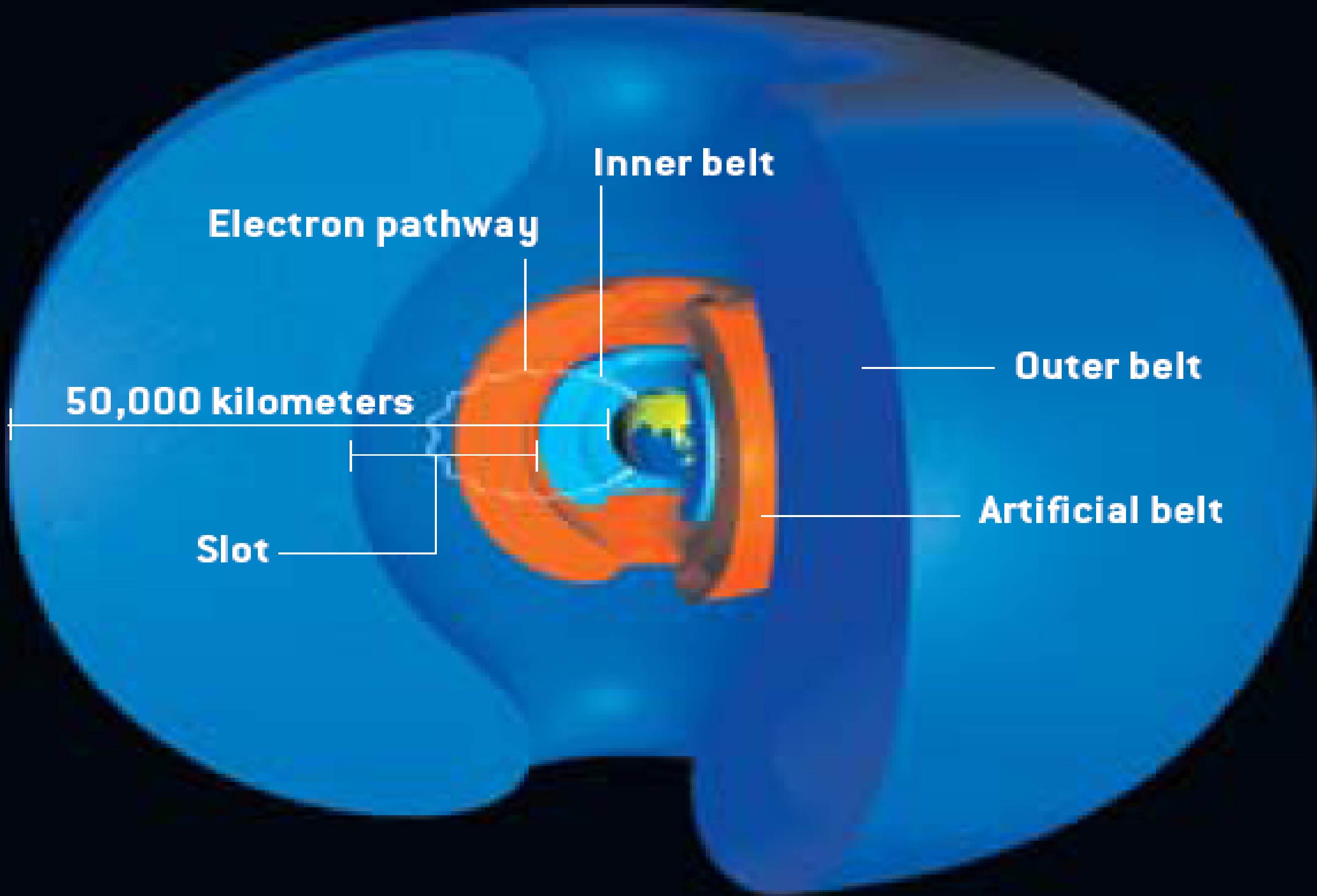
# VAN ALLEN RADIATION BELTS

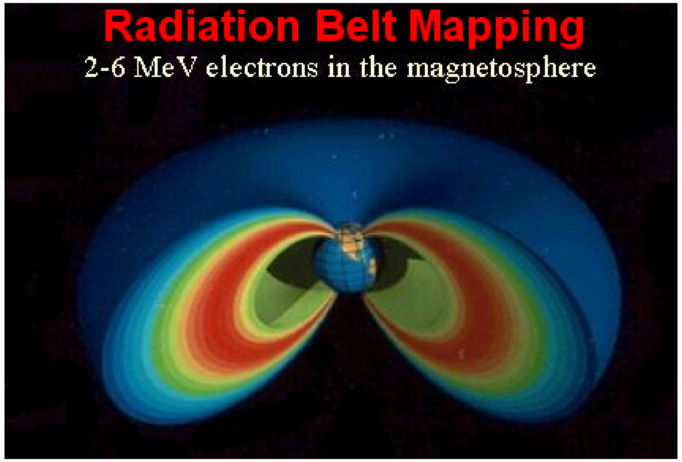
**Inner electron belt:**
**600 to 5,000 kilometers**

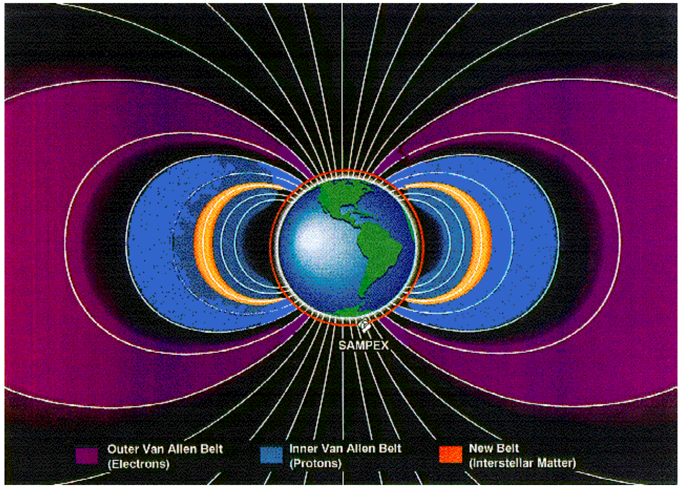
**Slot (safety zone):**
**6,000 to 12,000 kilometers**

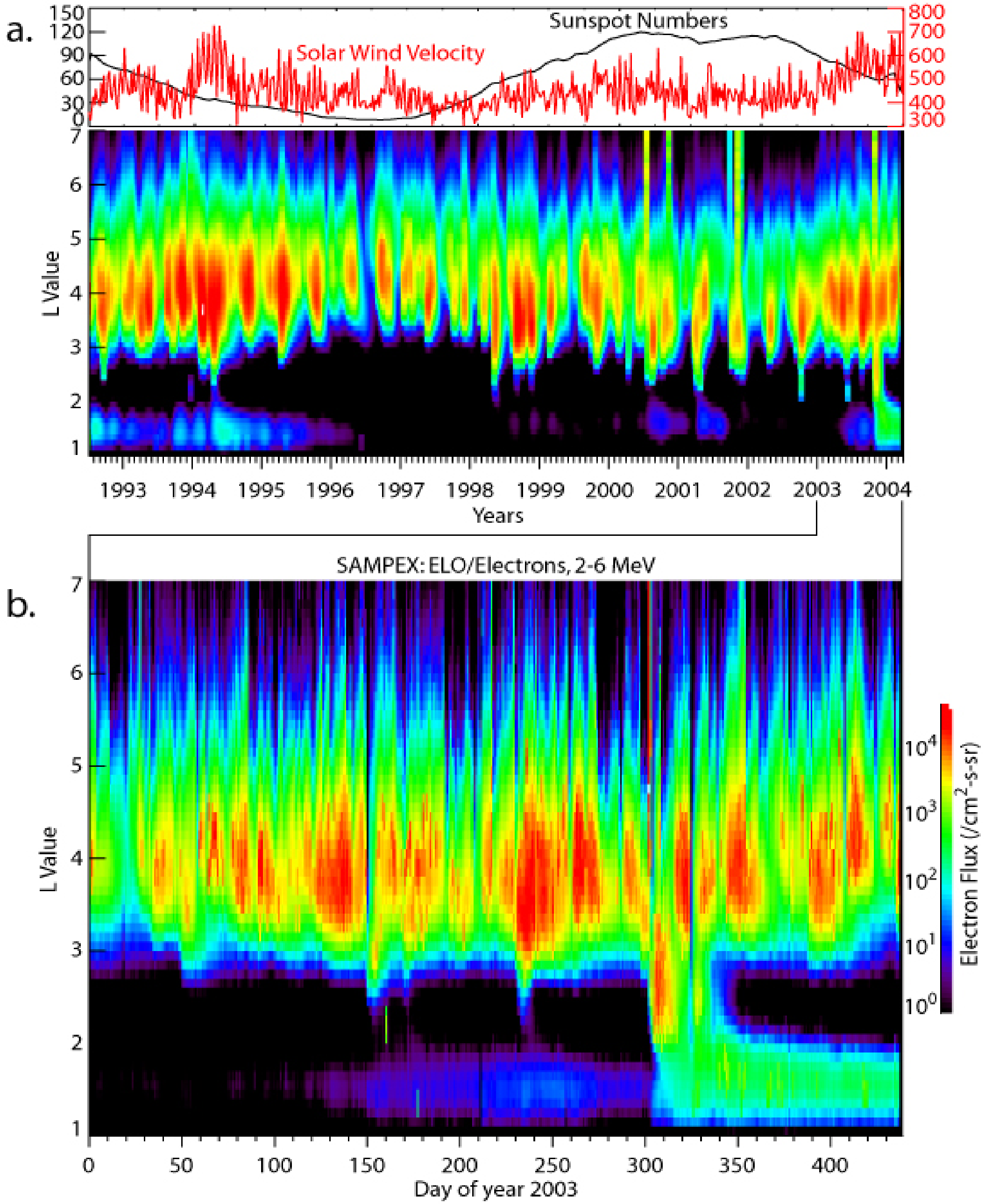
**Outer electron belt:**
**20,000 to 50,000 kilometers**



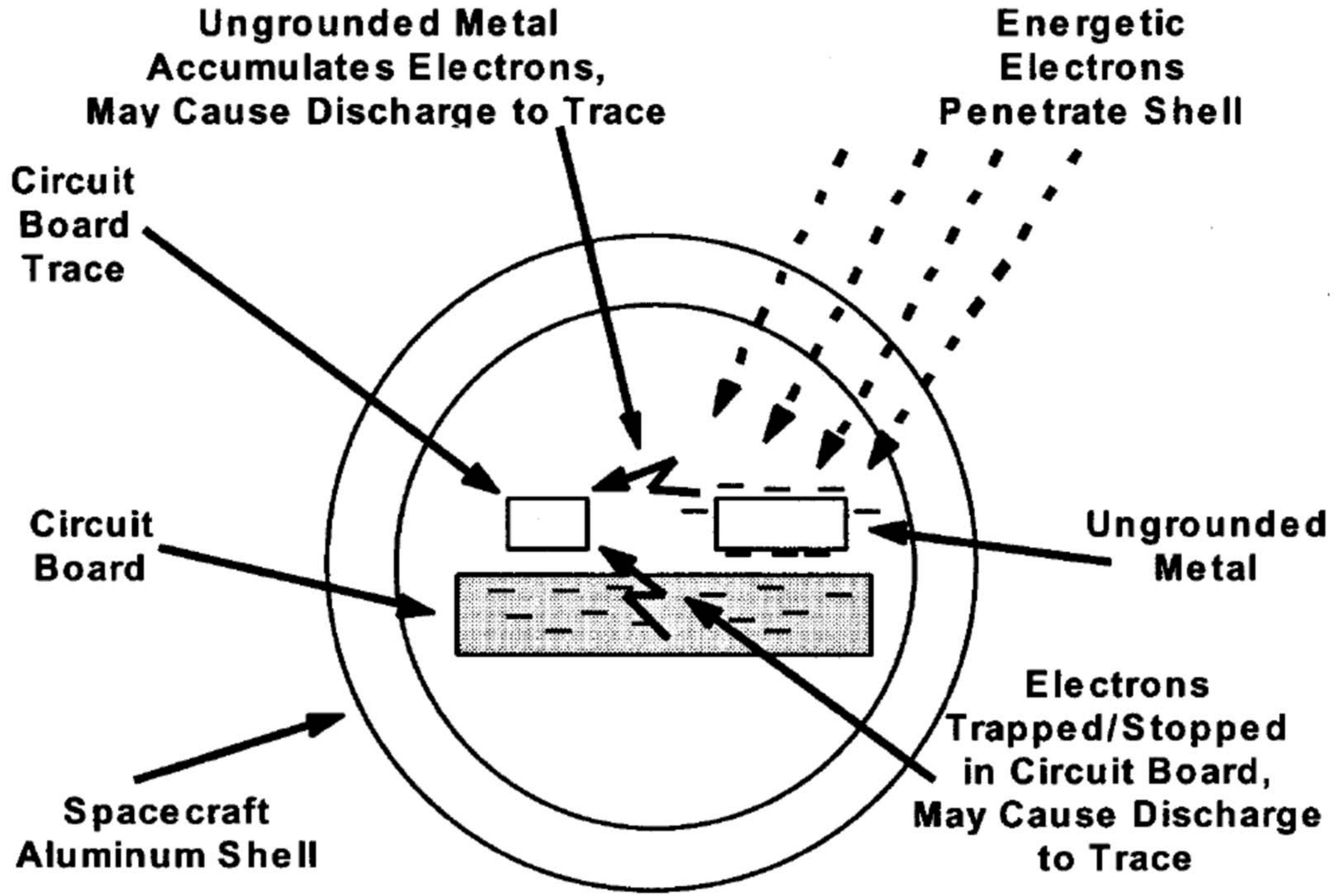


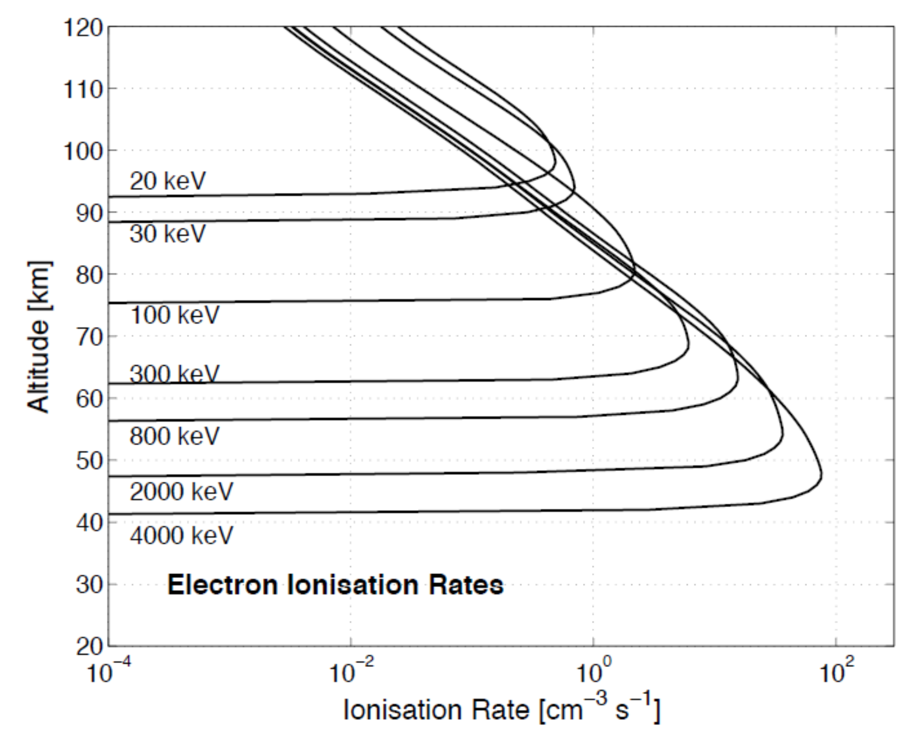

Radiation belt changes during Halloween storms: Baker et al. (*Nature, 2004)*

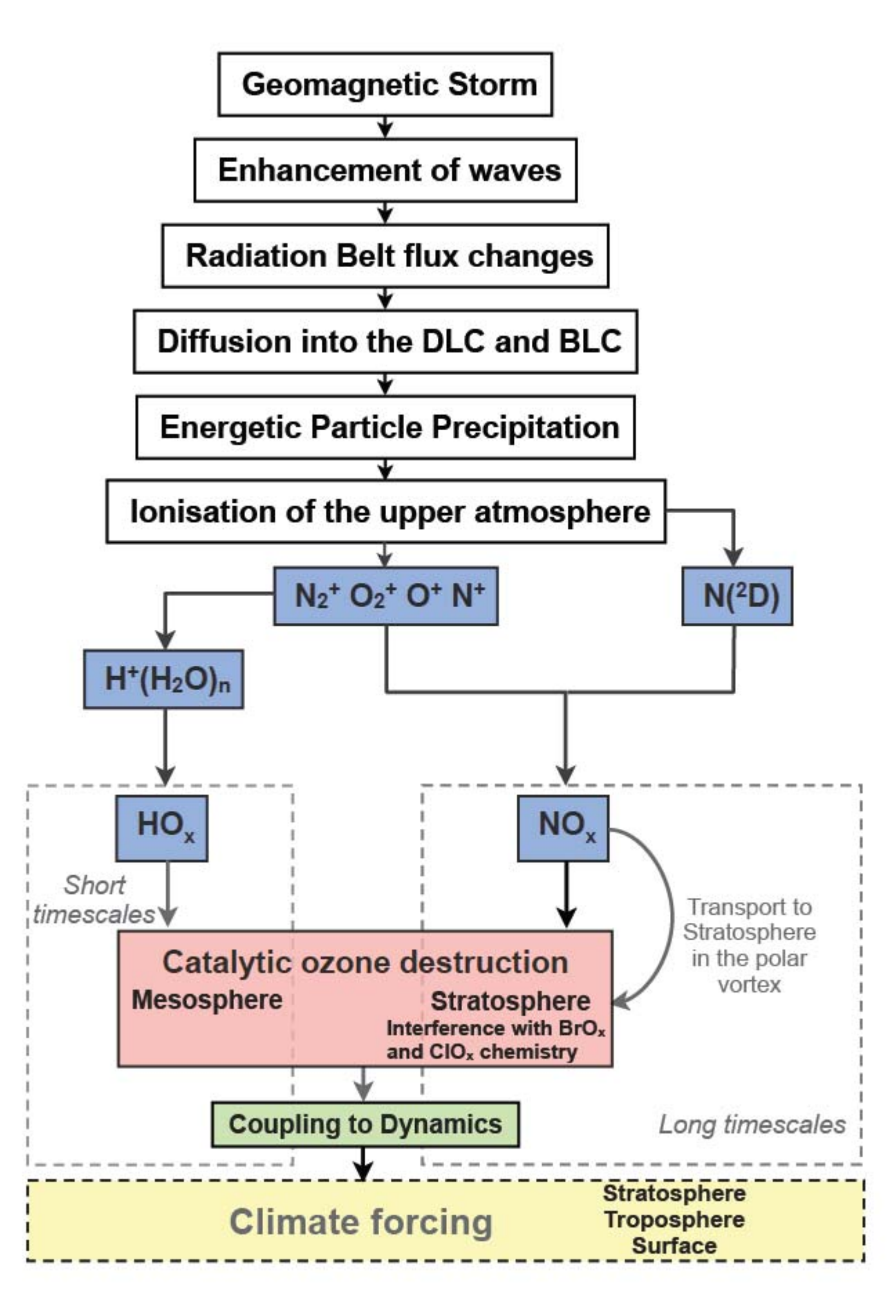
# Internal Charging of Spacecraft




NASA-HDBK-4002

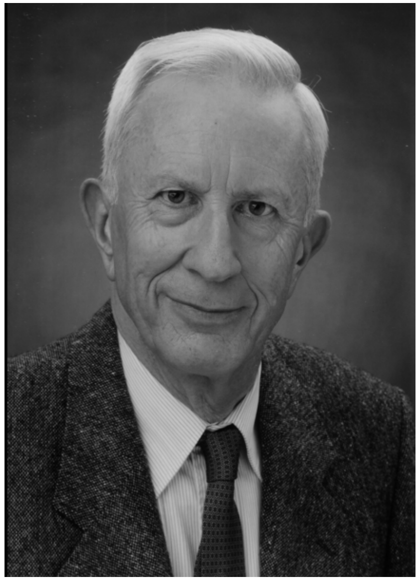
# Energetic Electron Precipitation into the Atmosphere



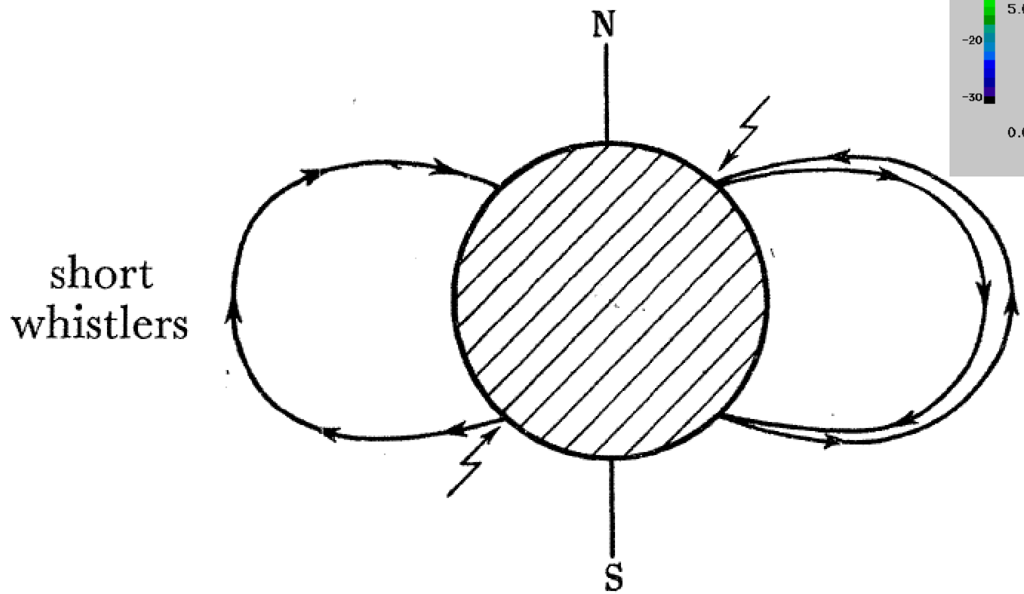



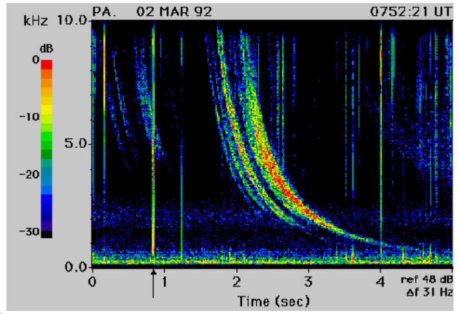

[*Clilverd et al.*, Oxford Univ. Press, 2015]

# Whistling Atmospherics



[L. R. O. Storey, 1953 ]

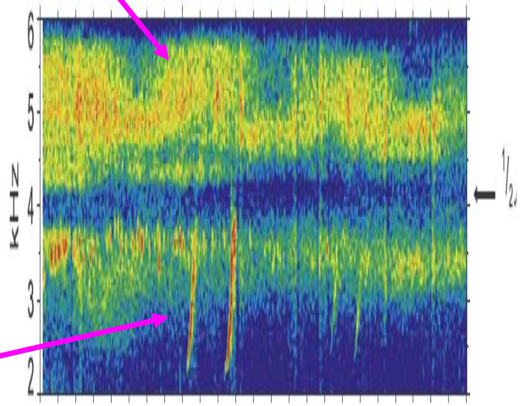



[Stanford Web]

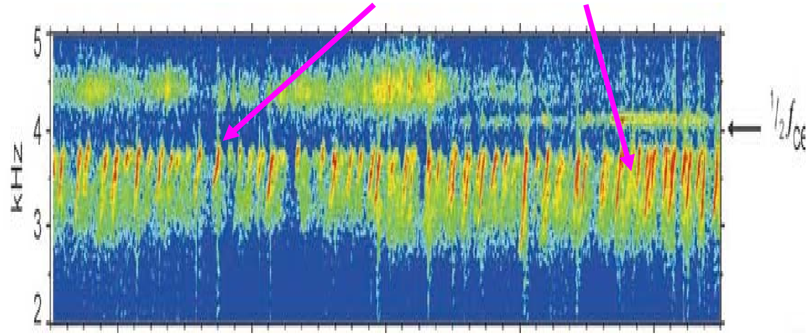
Outer Ionosphere = Magnetosphere

Hiss




Isolated Rising Whistles

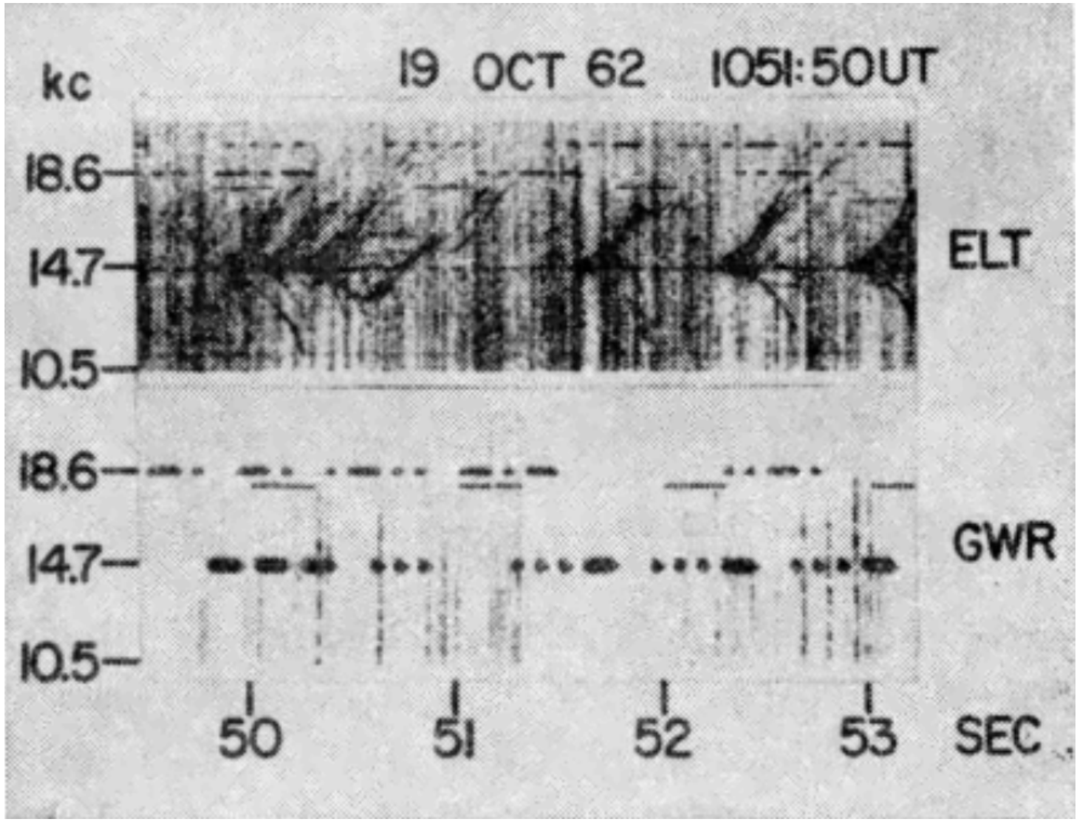
Dawn Chorus




[Santolik et al.,JGR, 2003]

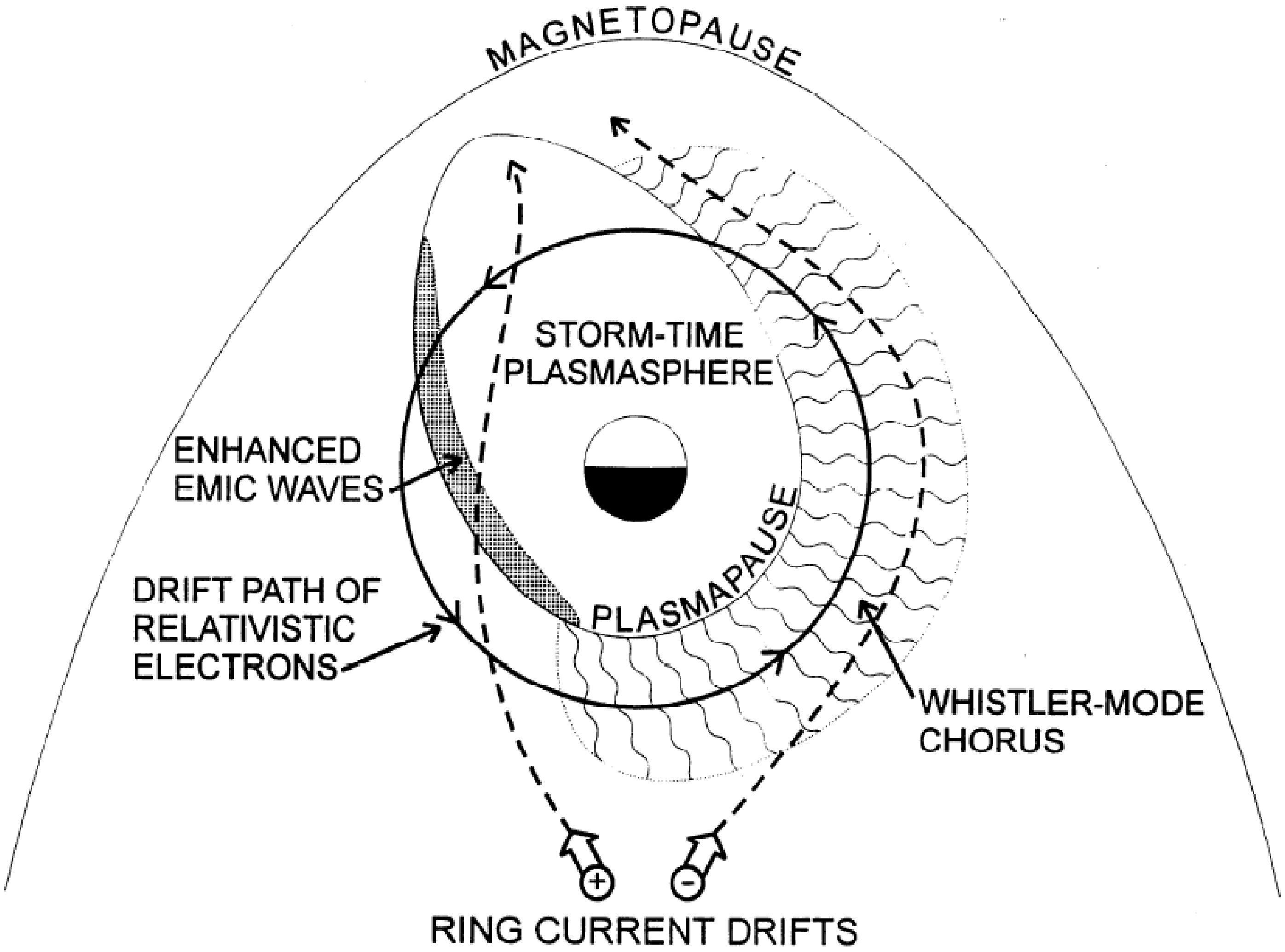
# VLF Triggered Emissions

[R. A. Helliwell, et al., JGR, 1964]




Rising and falling tones from the Morse code dashes

# Relativistic Theory of Wave-Particle Resonant Diffusion with Application to Electron Acceleration in the Magnetosphere



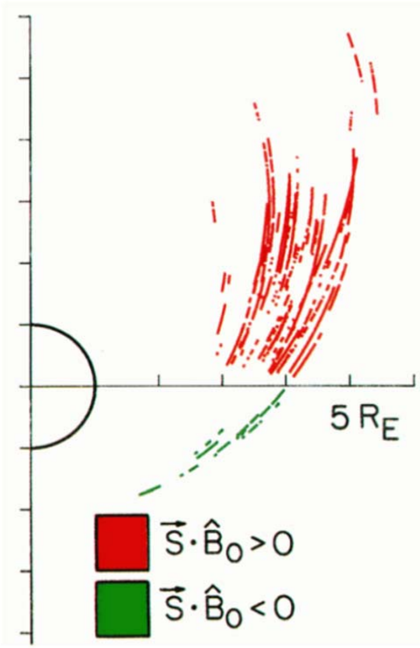

[*Summers et al., JGR, 1998*]

# New Generation of Spacecraft Observation

## Polar spacecraft

Source very close to Geomagnetic Equator

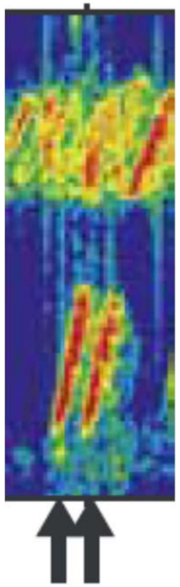
Absolute Instability at the Equator

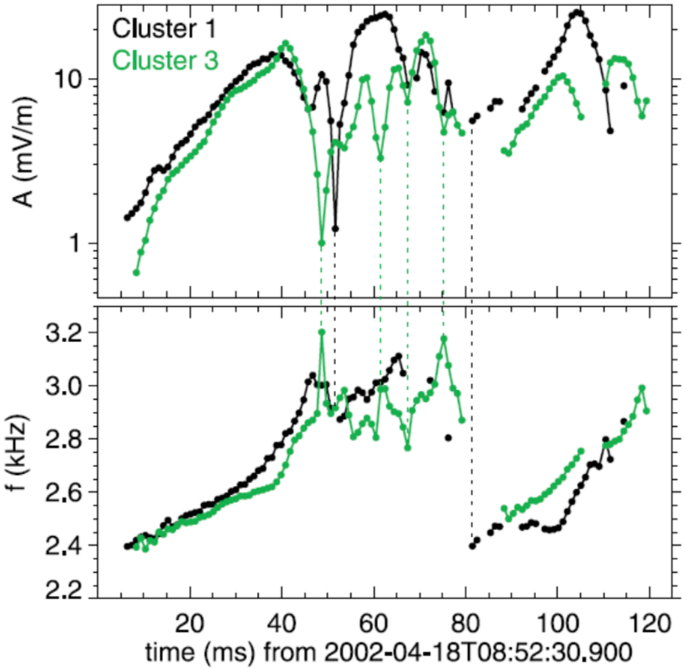



[LeDocq et al., 1998]

## Cluster spacecraft

Large Wave Amplitude:  10-100pt






Nonlinear Wave-Particle Interaction

[*Santolik et al., JGR, 2003*]

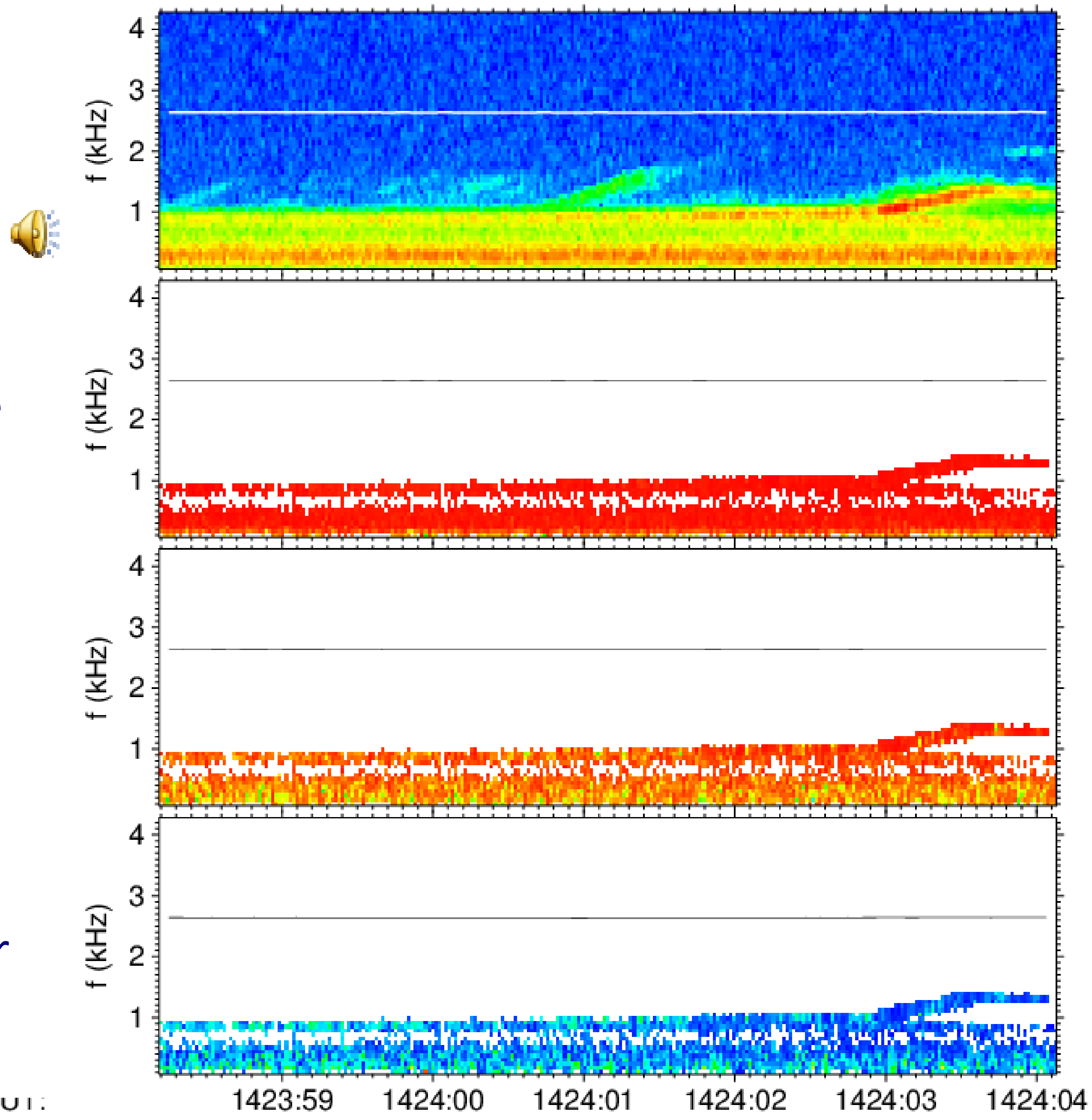
# EMFISIS Waves, Van Allen Probe B, 2 July 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



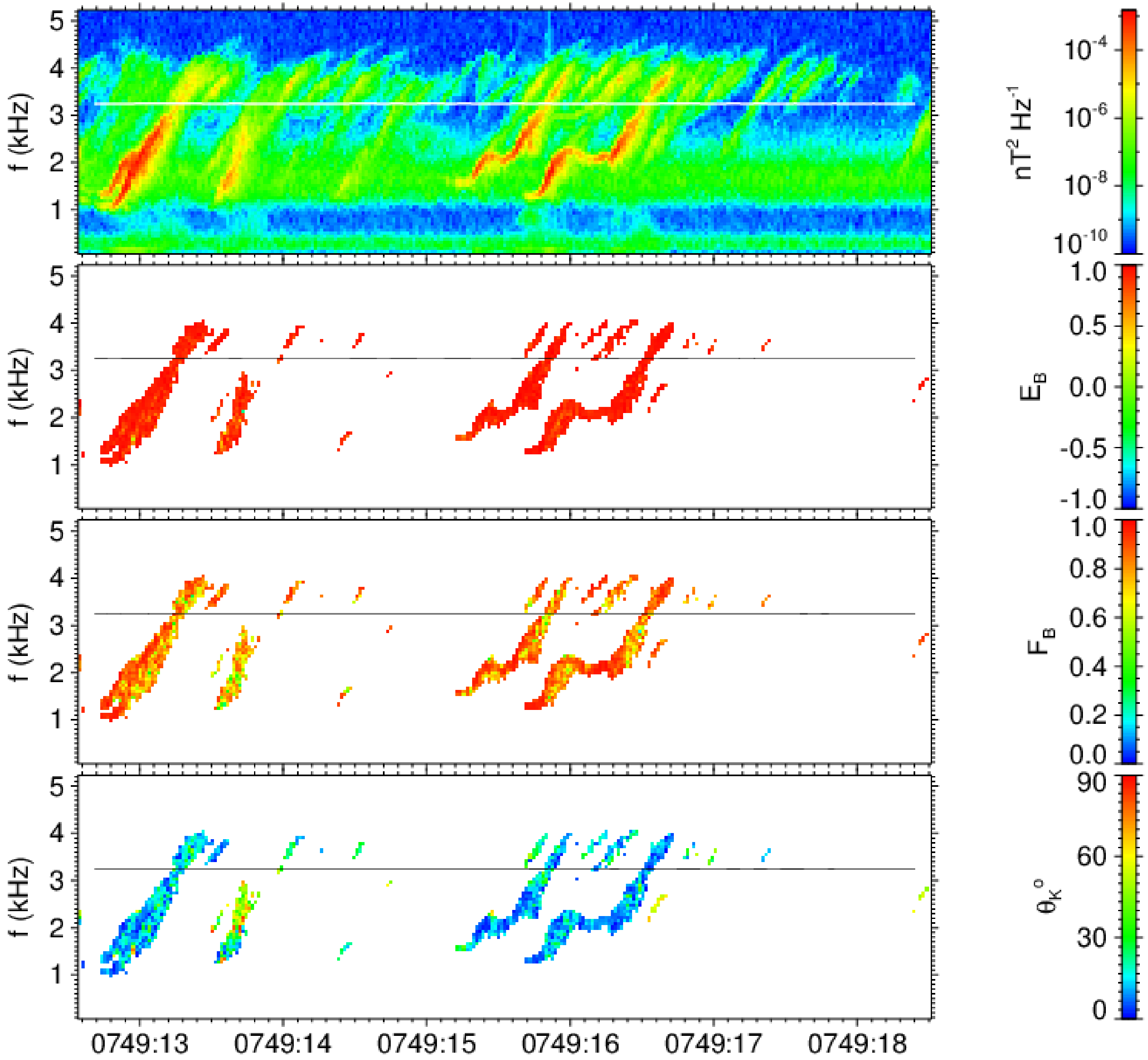
# EMFISIS Waves, Van Allen Probe A, 14 April 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



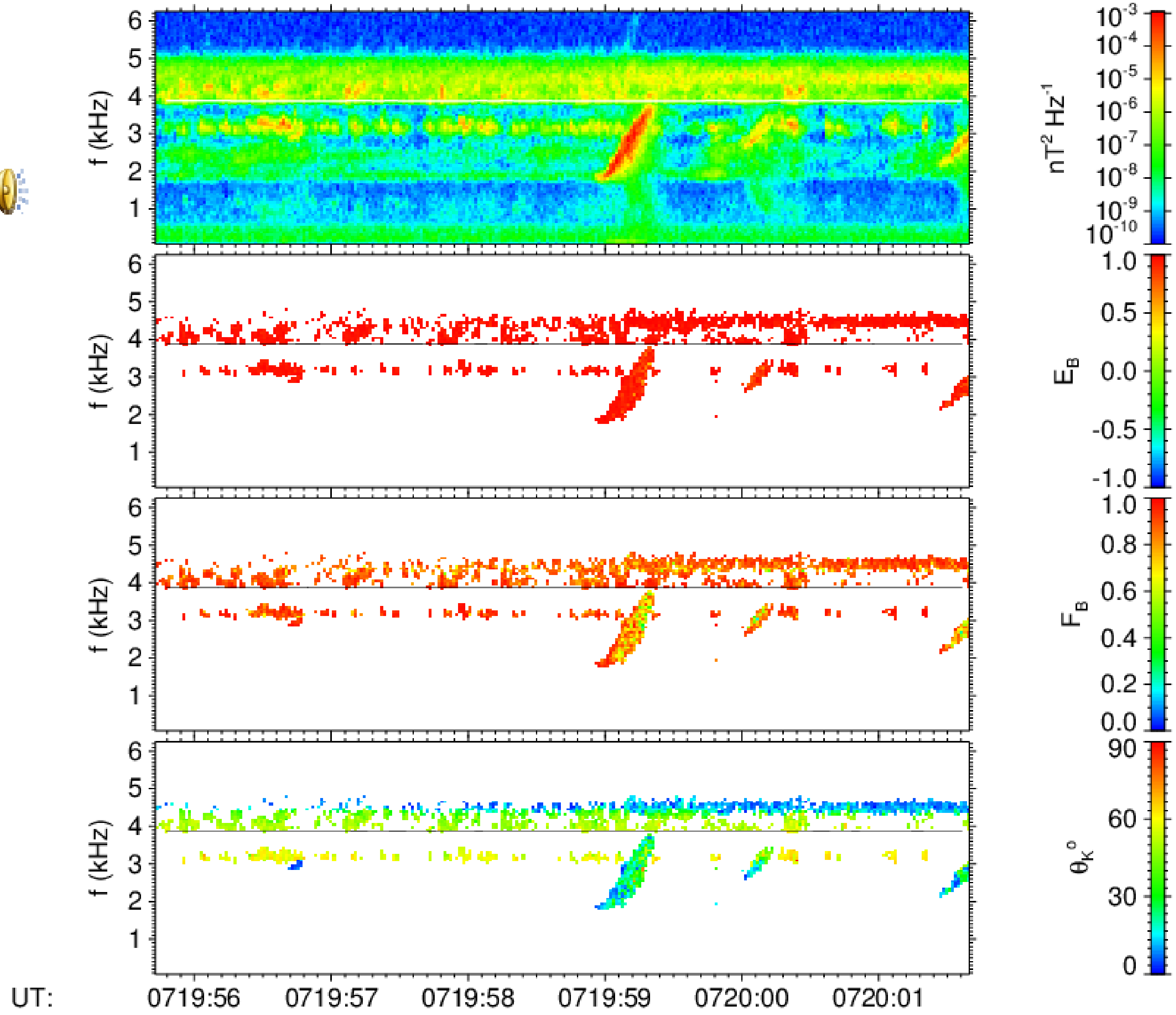
# EMFISIS Waves, Van Allen Probe A, 8 June 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



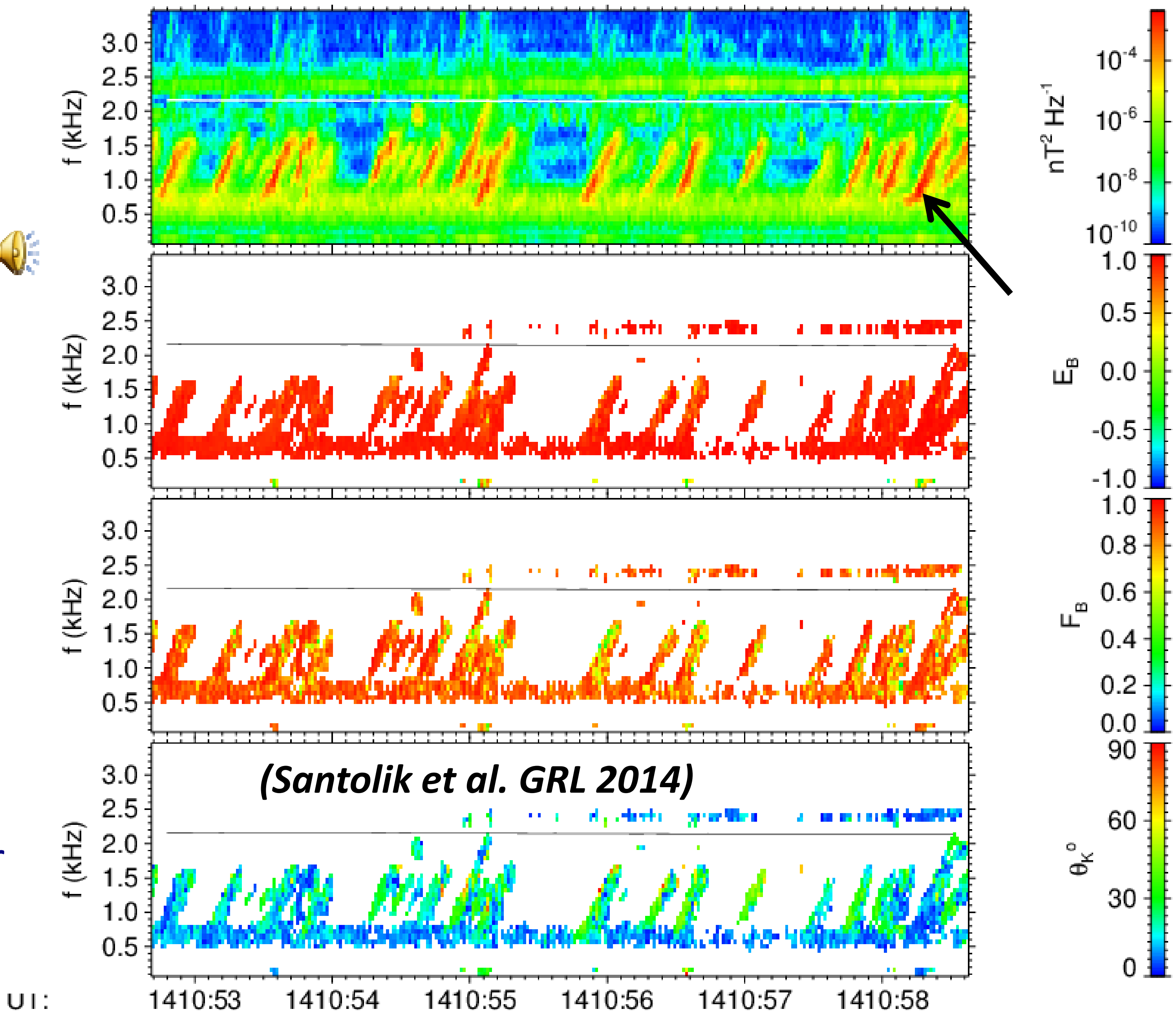
# EMFISIS Waves, Van Allen Probe A, 14 Nov 2012

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

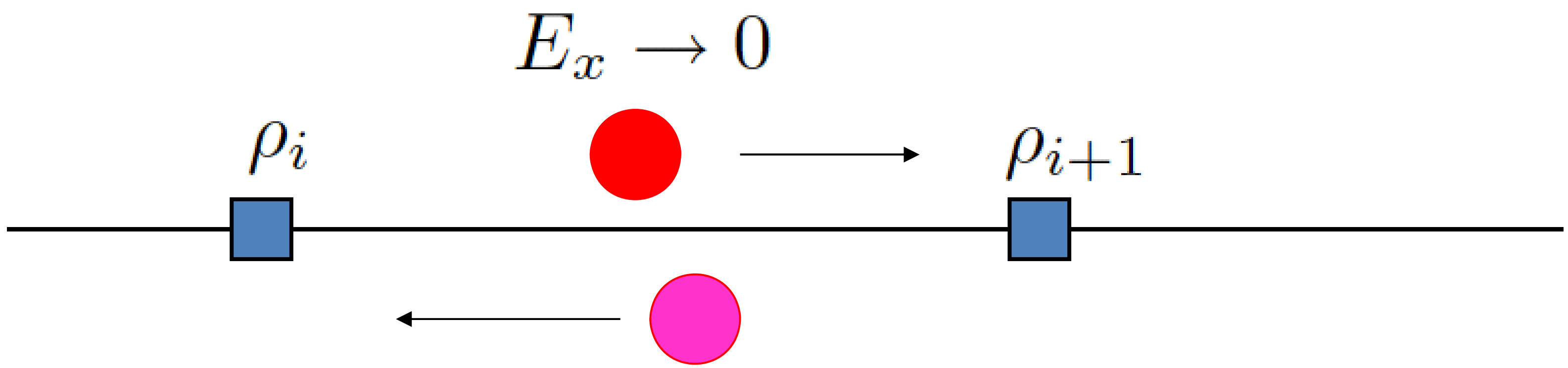
angle between the wave vector and the background magnetic field



# PIC code for Space Plasmas

- **Space Plasmas**: **Collisionless**
- **Particle-In-Cell Code**
- **Particles**: x(t), **v**(t)
- **Fields**: ***E***(t, X), ***B***(t, X)

  ***E*** and ***B*** are defined on grid points, and calculated from ρ and ***J***. The electrostatic force between two particles in the same cell disappears.



# Maxwell's Equations

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

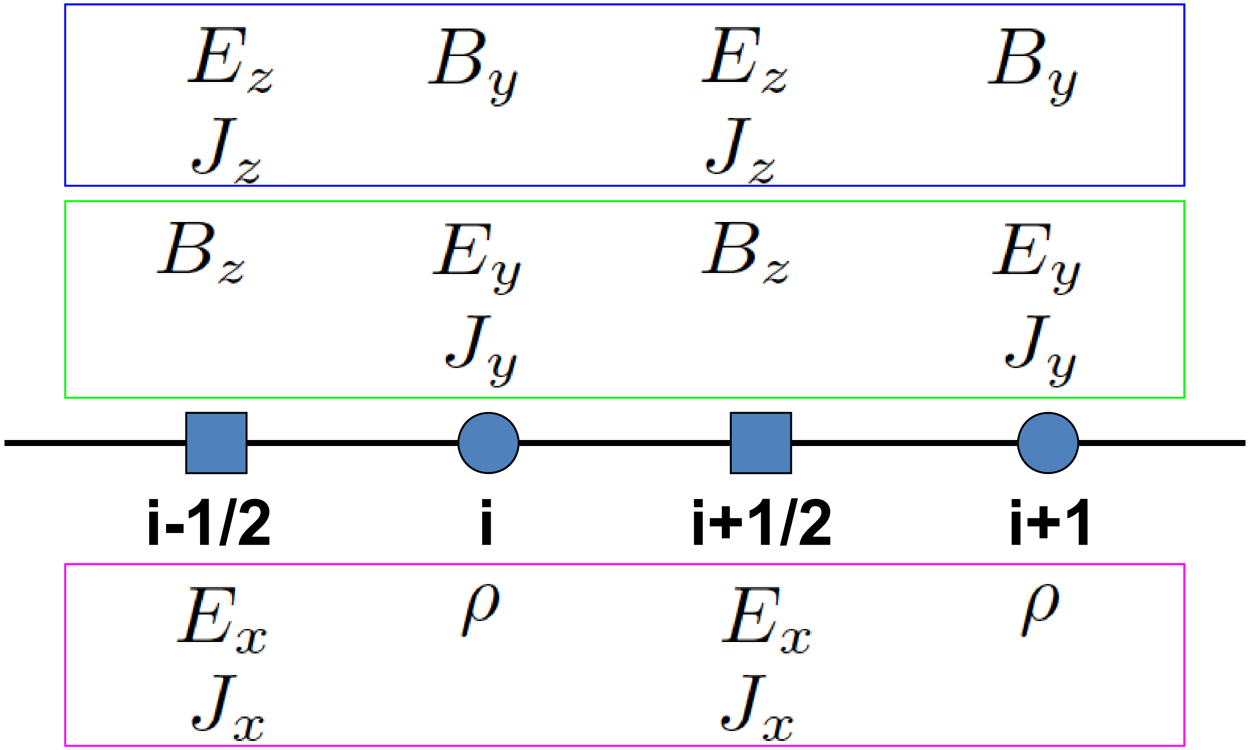
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

where $\varepsilon_0 \mu_0 = \frac{1}{c^2}$

# Grid Assignment




$$\frac{\partial E_z}{\partial t} = c^2 \frac{\partial B_y}{\partial x} - J_z$$

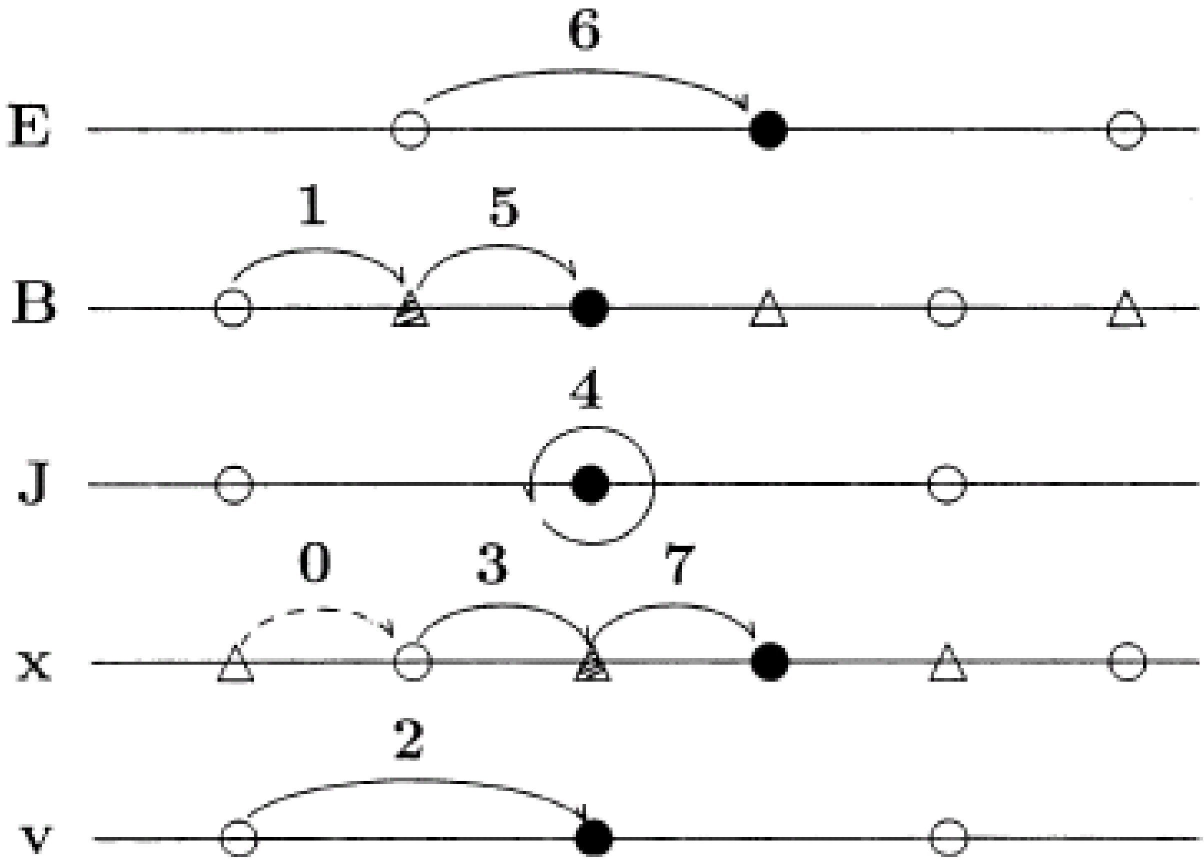
$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$\frac{\partial E_y}{\partial t} = -c^2 \frac{\partial B_z}{\partial x} - J_y$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial E_x}{\partial t} = -J_x$$

# Time Step Chart



# Centered Difference Scheme

$$E(X_i, t) = E_o \exp(ikX_i - i\omega t)$$

$$\frac{\partial E(X_i,t)}{\partial x} = \frac{E(X_i + \Delta x/2, t) - E(X_i - \Delta x/2, t)}{\Delta x}$$

$$= \frac{1}{\Delta x}\left[\exp(ik\Delta x/2) - \exp(-ik\Delta x/2)\right] E(X_i, t)$$

$$= i\frac{\sin(k\Delta x/2)}{\Delta x/2} E(X_i, t) = iKE(X_i, t)$$

$$k \Longrightarrow K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

$$\omega \Longrightarrow \Omega = \frac{\sin(\omega\Delta t/2)}{\Delta t/2}$$

# Courant Condition

Electromagnetic modes in vacuum

$$\omega^2 = c^2 k^2$$

Centered Difference Scheme in space and time

$$\Omega^2 = c^2 K^2 \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

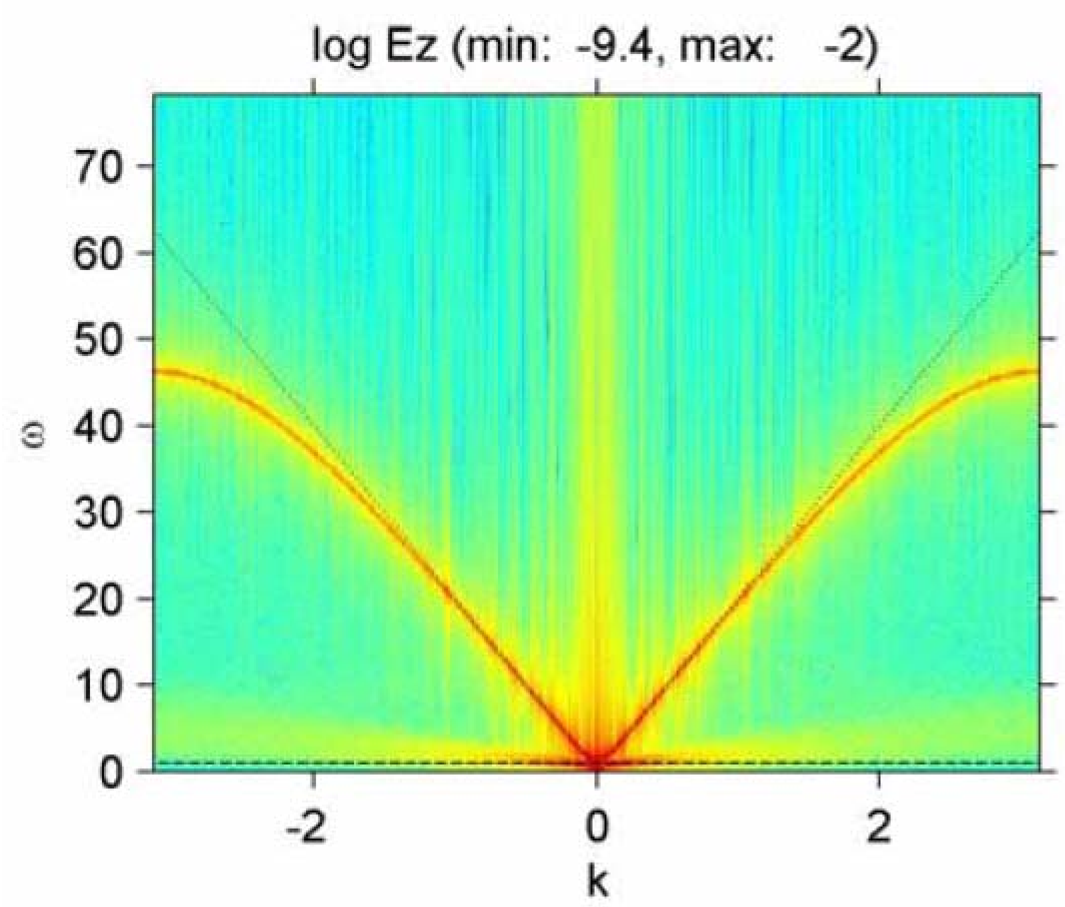
For $k = \dfrac{\pi}{\Delta x}$ we have $\sin(\dfrac{\omega \Delta t}{2}) = \dfrac{\Delta t}{\Delta x} c < 1$

Courant Condition

$$\boxed{c\Delta t < \Delta x}$$

# Dispersion Relation of Light Mode

$$\Omega^2 = c^2 K^2$$




$$\Omega = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \qquad K = \frac{\sin(k\Delta x/2)}{\Delta x/2}$$

# Charge Density

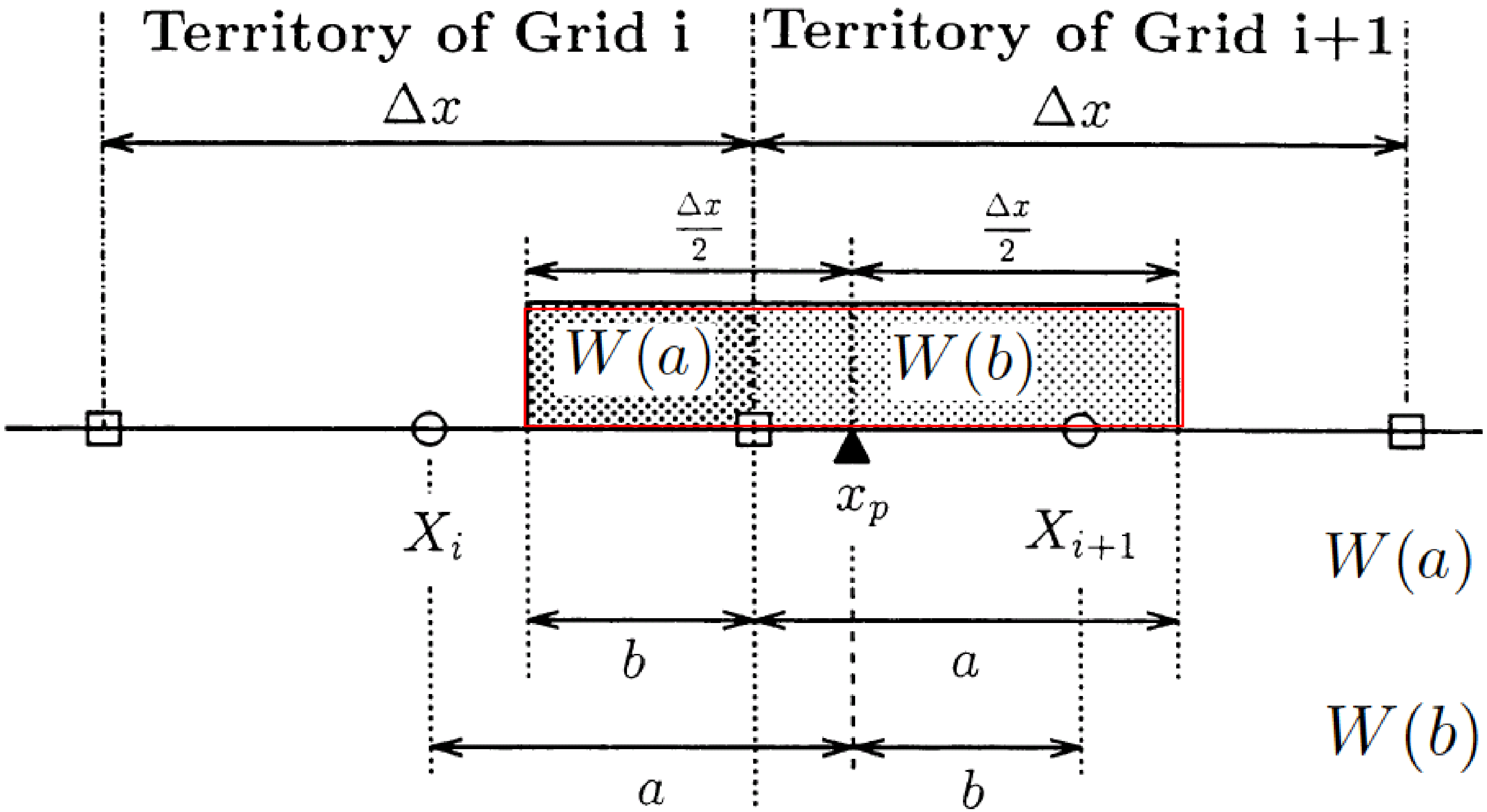
$$\rho_i = \frac{1}{\Delta x} \sum_{j}^{N_p} q_j W(x_j - X_i)$$

Np: Number of Particles

## Shape Function

$$W(x) = 1 - \frac{|x|}{\Delta x}, \quad |x| \leq \Delta x$$
$$= 0, \quad |x| > \Delta x$$

**“fat particle”**




$$W(a) = \frac{b}{\Delta x}$$

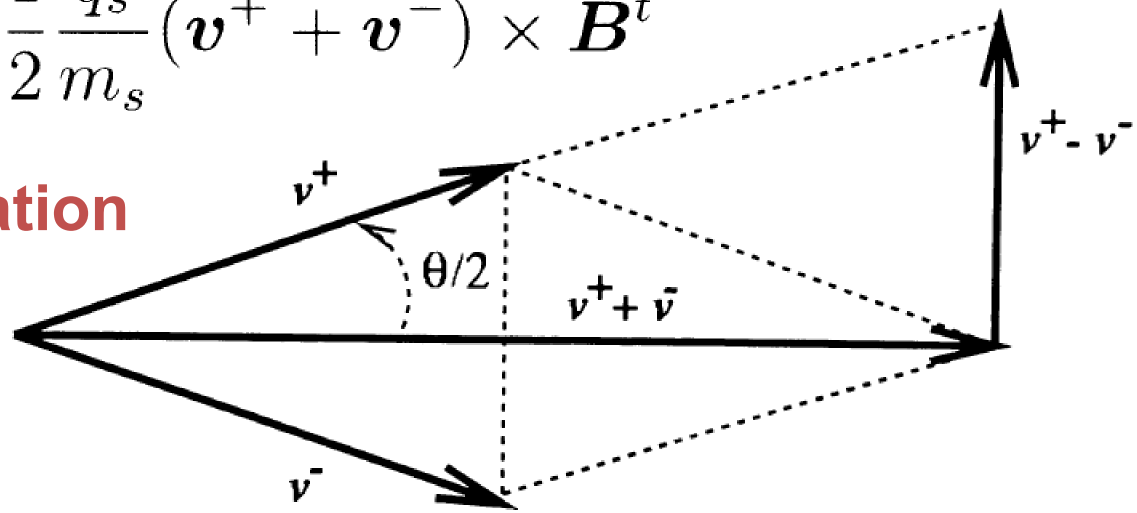
$$W(b) = \frac{a}{\Delta x}$$

# Buneman-Boris Method

$$\frac{\boldsymbol{v}^{t+\Delta t/2} - \boldsymbol{v}^{t-\Delta t/2}}{\Delta t} = \frac{q_s}{m_s}\left(\boldsymbol{E}^t + \frac{\boldsymbol{v}^{t+\Delta t/2} + \boldsymbol{v}^{t-\Delta/2}}{2} \times \boldsymbol{B}^t\right)$$

$$\boldsymbol{v}^- = \boldsymbol{v}^{t-\Delta t/2} + \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2} \qquad \boldsymbol{v}^+ = \boldsymbol{v}^{t+\Delta t/2} - \frac{q_s}{m_s}\boldsymbol{E}^t\frac{\Delta t}{2}$$

$$\frac{\boldsymbol{v}^+ - \boldsymbol{v}^-}{\Delta t} = \frac{1}{2}\frac{q_s}{m_s}(\boldsymbol{v}^+ + \boldsymbol{v}^-) \times \boldsymbol{B}^t$$




**Kinetic Energy Conservation**

$$(\boldsymbol{v}^+)^2 = (\boldsymbol{v}^-)^2$$

**Small Phase Delay**

$$\Omega_c = \frac{\tan^{-1}\omega_c\Delta t/2}{\Delta t/2}$$

$$\Omega_c/\omega_c = 0.9967 \text{ with } \omega_c\Delta t = 0.2$$

# Relativistic Equation of Motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\boldsymbol{v}) = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\boldsymbol{u} = \frac{c}{\sqrt{c^2 - |\boldsymbol{v}|^2}}\boldsymbol{v}$$

$$\boldsymbol{B}_u = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{B}$$

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}_{\boldsymbol{u}}\right)$$

$$\boldsymbol{v} = \frac{c}{\sqrt{c^2 + |\boldsymbol{u}|^2}}\boldsymbol{u}$$

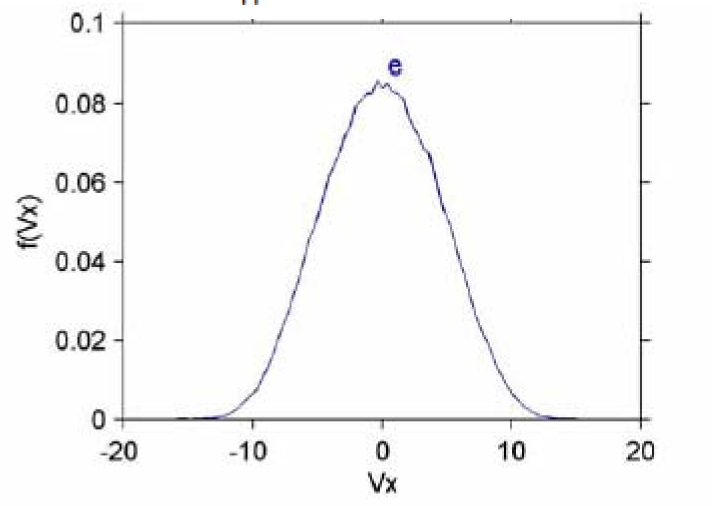
# Initial Velocity Distribution Function

$$f(u_{||}, u_{\perp}) \propto \exp(-\frac{(u_{||} - V_{d||})^2}{2V_{t||}^2} - \frac{(u_{\perp} - V_{d\perp})^2}{2V_{t\perp}^2})$$

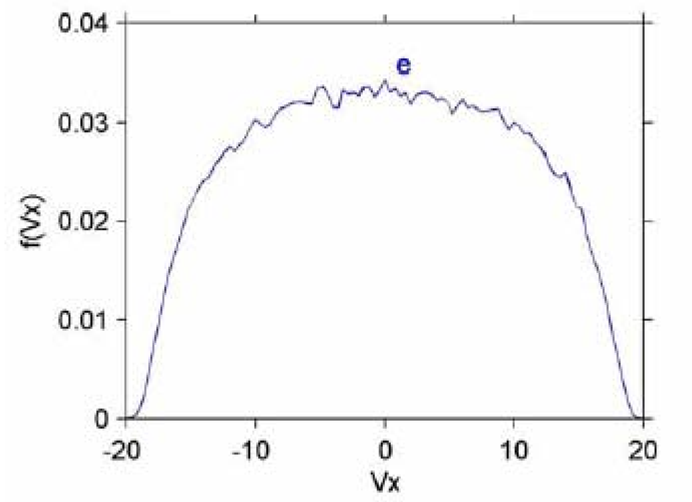
$$\boldsymbol{v} = \boldsymbol{u}/\gamma = \frac{c}{\sqrt{c^2 + u_x^2 + u_y^2 + u_z^2}}\boldsymbol{u}$$

$$V_{d||} = V_{d\perp} = 0$$

$$V_{t||}/c = 0.25$$




$$V_{t||}/c = 1$$

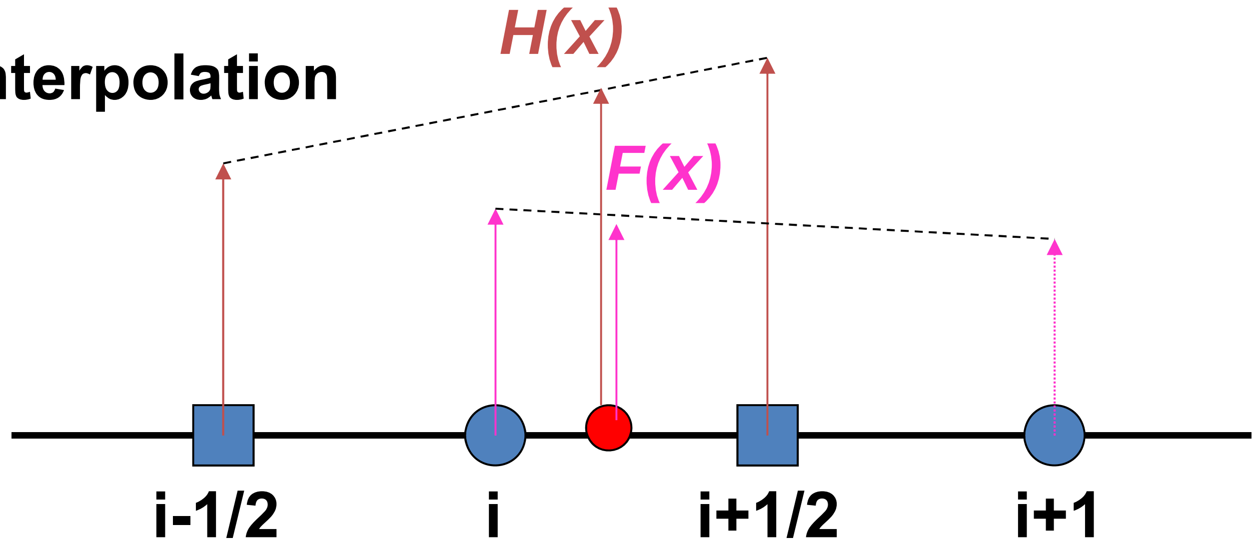


# Field Interpolation to Particle Position

$$F(x) = \sum_{i=1}^{N_x} \boxed{F_i} W(x - X_i)$$

$$H(x) = \sum_{i=1}^{N_x} \boxed{H_{i+1/2}} W(x - X_{i+1/2})$$

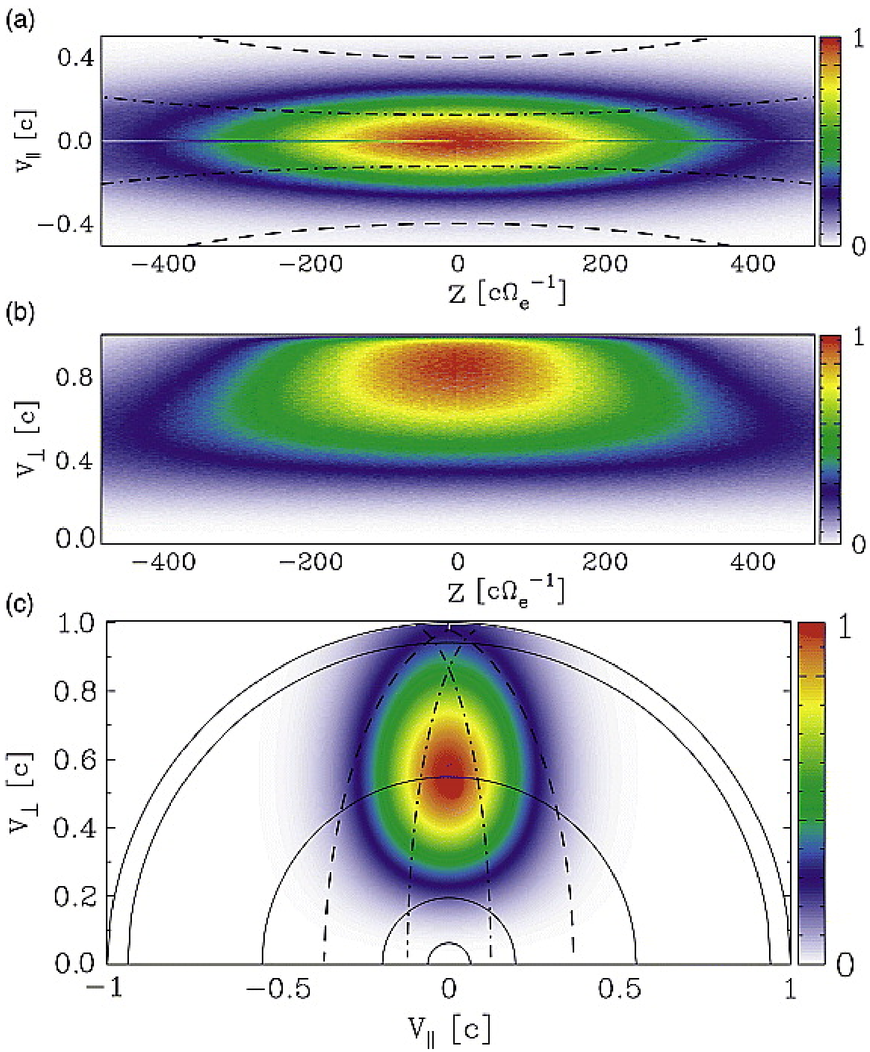
**Linear Interpolation**

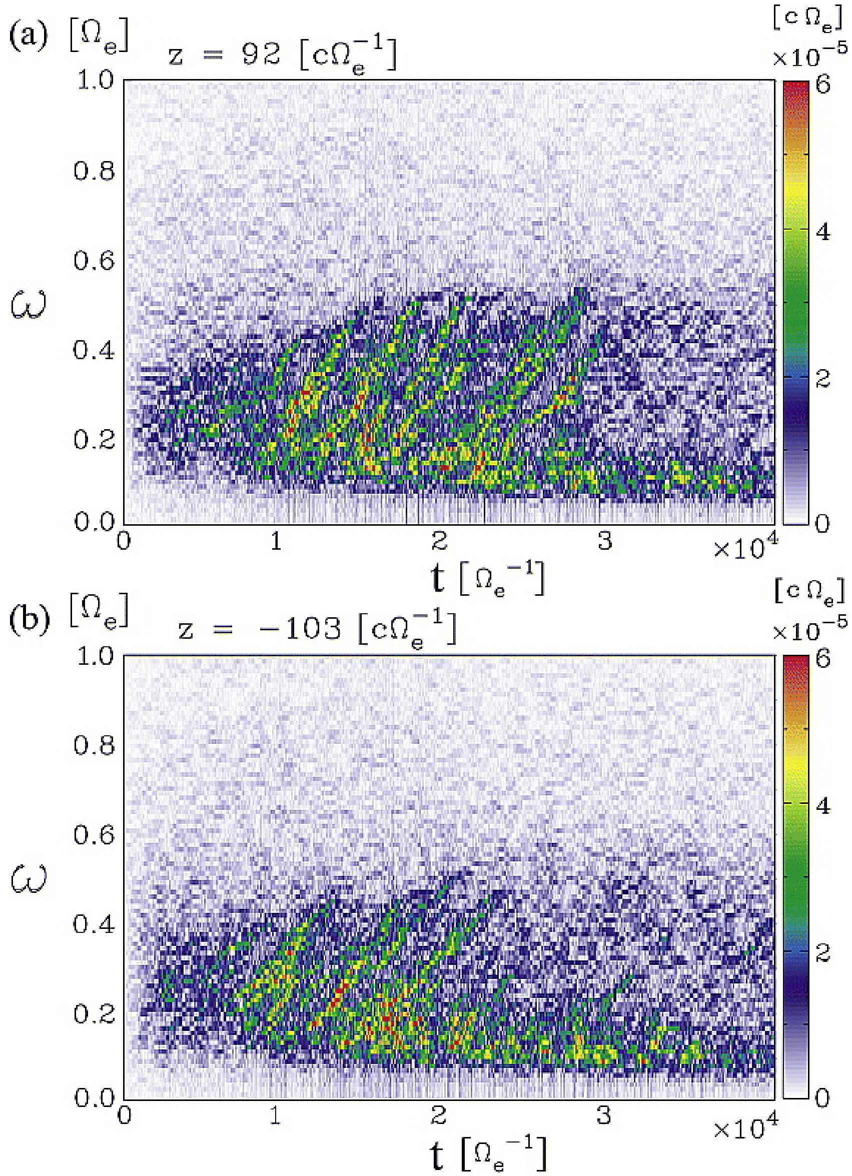


# Electron Hybrid Simulation

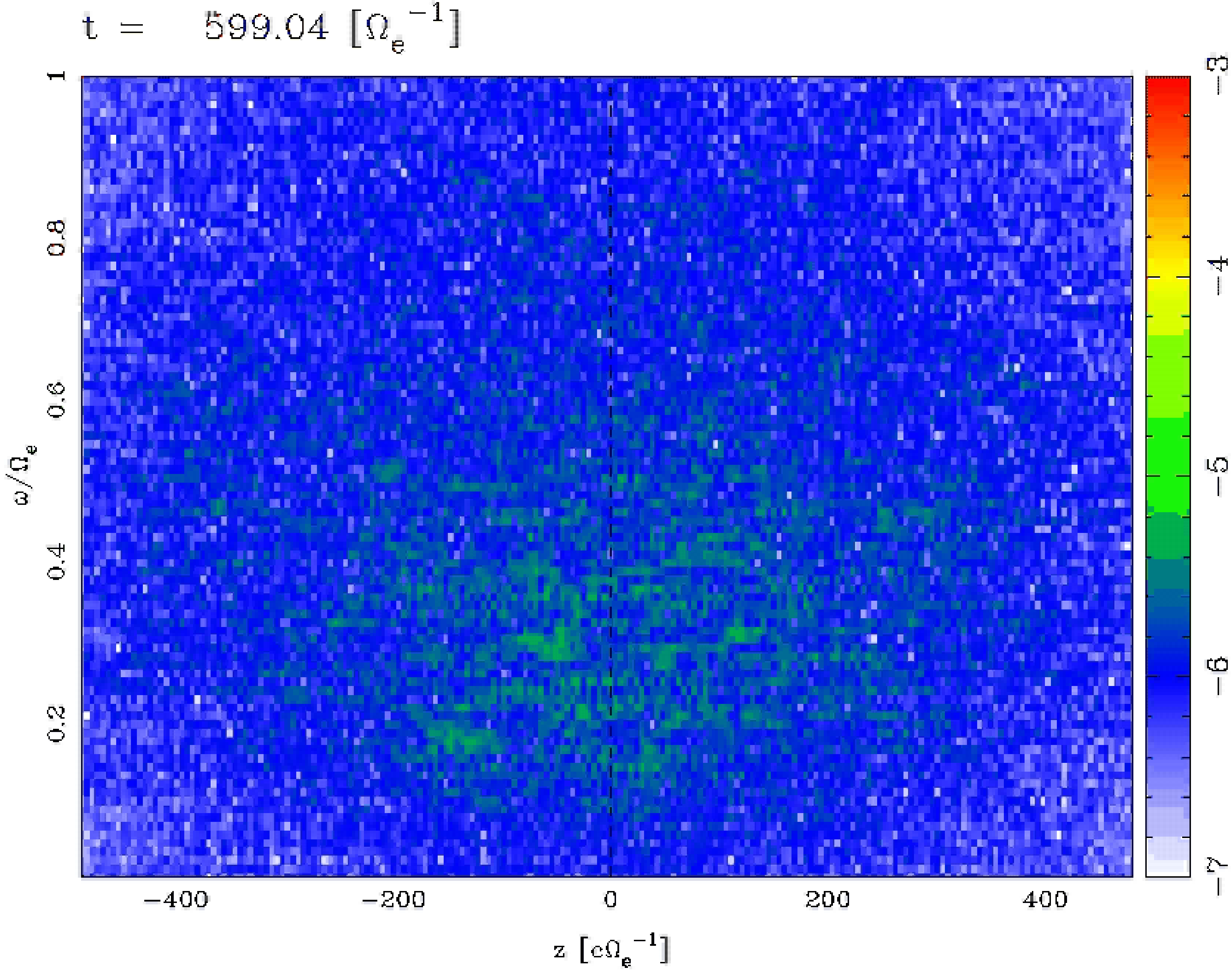
Cold electrons: Fluid

Hot electrons: Particles




[Katoh and Omura,GRL, 2007]



t = 599.04 [Ω_e^-1]
ω/Ω_e
1
0.8
0.6
0.4
0.2
−400
−200
0
200
400
z [cΩ_e^-1]
−3
−4
−5
−6
−7

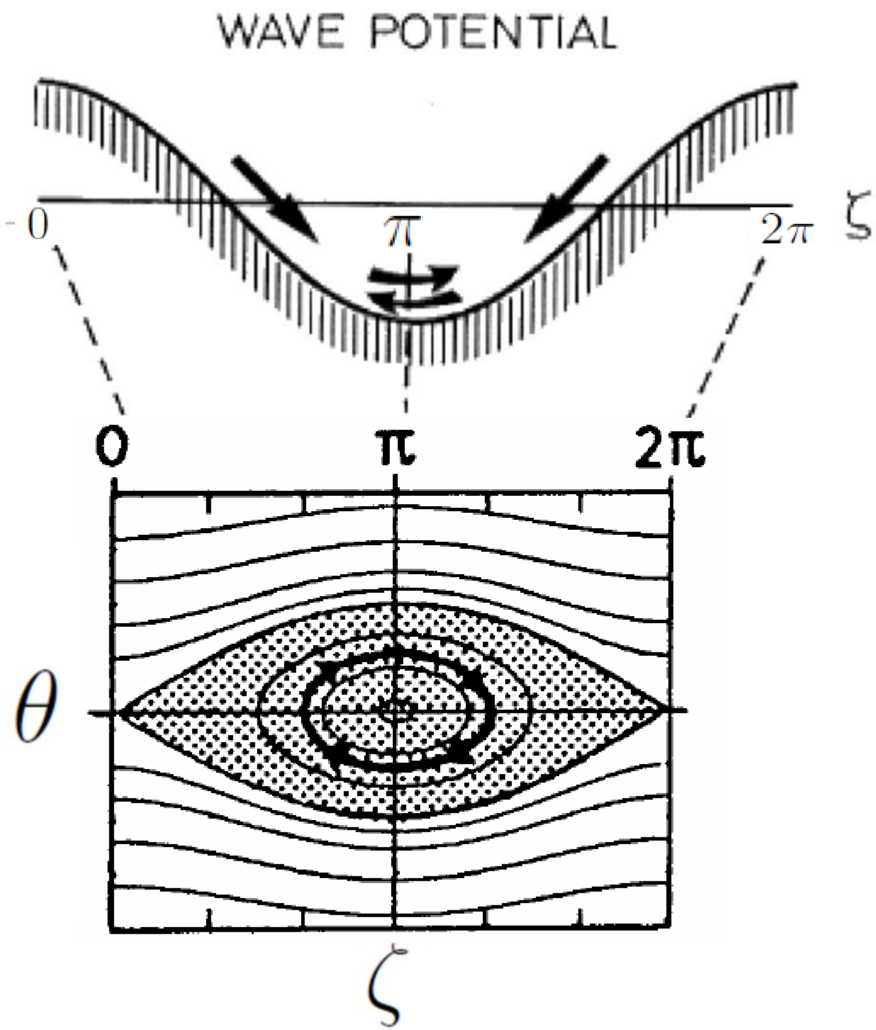
Equations of Resonant Particles

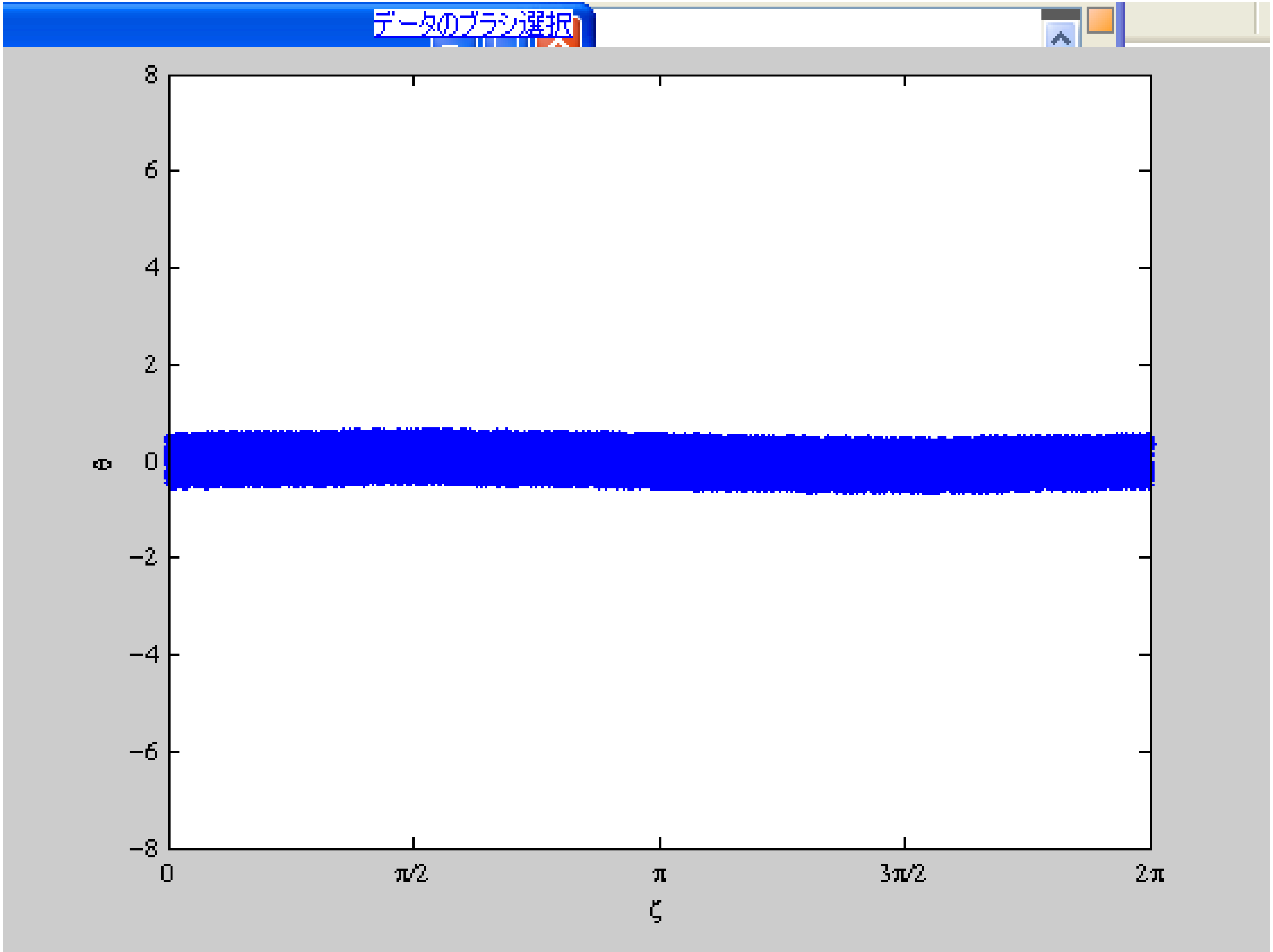
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_t^2 \sin\zeta$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \theta$$

Trapping Frequency

$$\omega_t = \sqrt{\frac{k|q_s|E_w}{m_s}}$$




データのブラシ選択
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ

$$\theta = k(v_{\parallel} - V_p)$$ for Longitudinal Wave

$$\theta = k(v_{\parallel} - V_R)$$ for Whistler-mode Wave

$$V_R = \frac{\omega - \Omega_e}{k}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_t^2(\sin\zeta + \boxed{S})$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \theta$$

$S$ : Inhomogeneity Factor

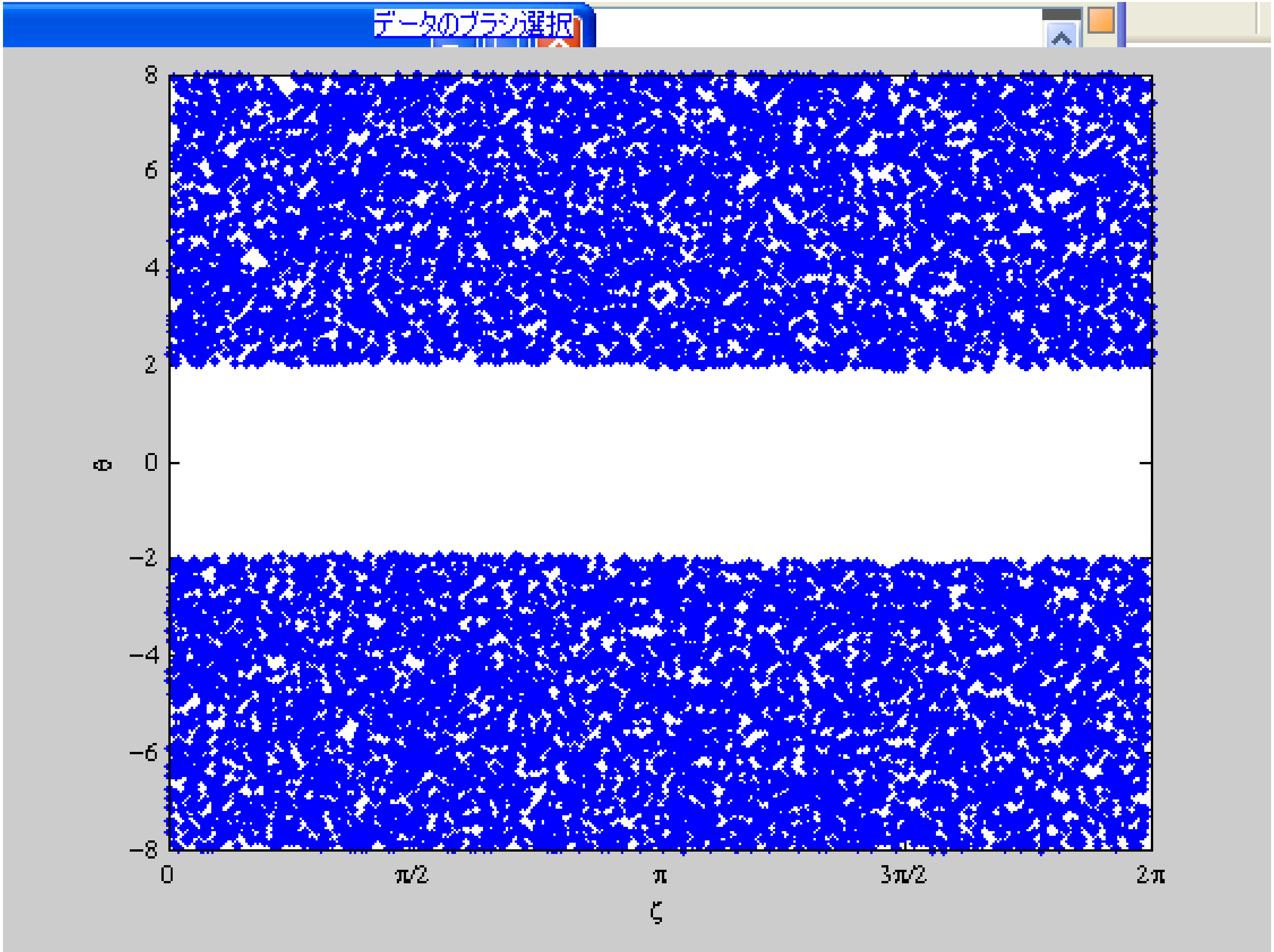
# Inhomogeneity Factor

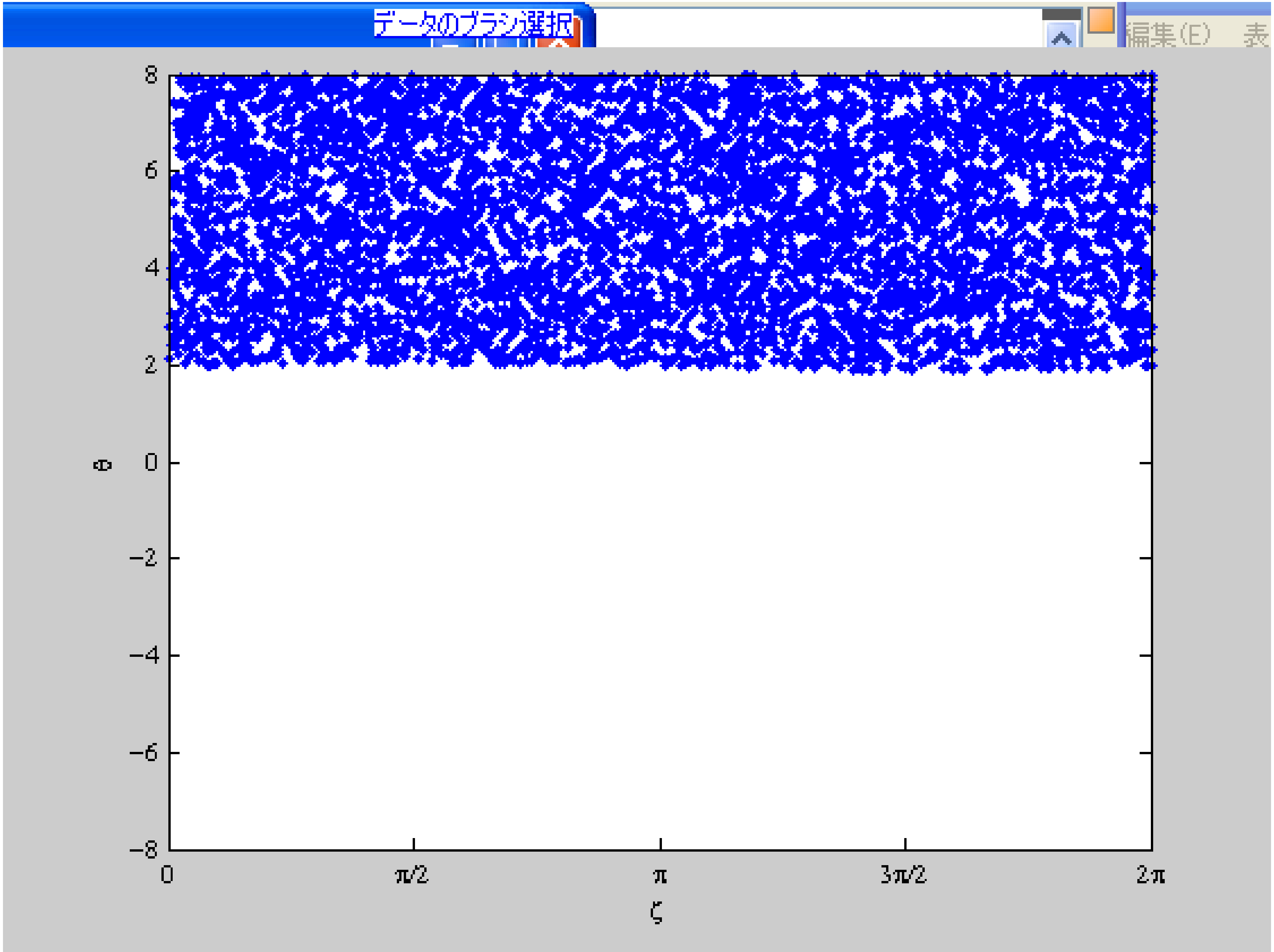
$$S = -\frac{1}{s_0 \omega \Omega_w} \left( s_1 \frac{\partial \omega}{\partial t} + c s_2 \frac{\partial \Omega_e}{\partial h} \right)$$

$$s_0 = \frac{\delta}{\xi} \frac{V_{\perp 0}}{c} \qquad s_1 = \gamma \left(1 - \frac{V_R}{V_g}\right)^2$$

$$s_2 = \frac{1}{2\xi\delta} \left\{ \frac{\gamma\omega}{\Omega_e} \left( \frac{V_{\perp 0}}{c} \right)^2 - \left[ 2 + \Lambda \frac{\delta^2 (\Omega_e - \gamma\omega)}{\Omega_e - \omega} \right] \frac{V_R V_p}{c^2} \right\}$$

[Omura et al., JGR, 2008; 2009]


データのブラシ選択
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ


データのブラシ選択
編集(E)
表
8
6
4
2
0
−2
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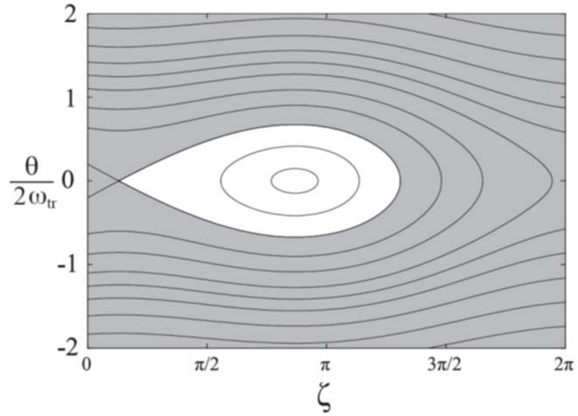
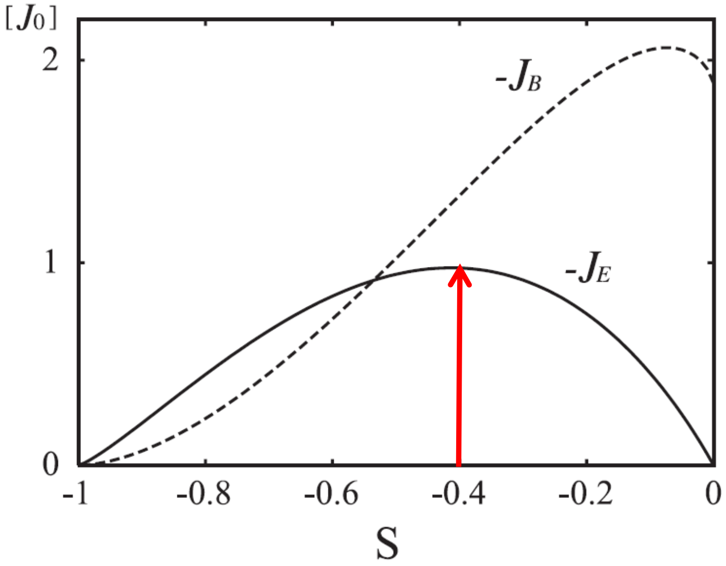
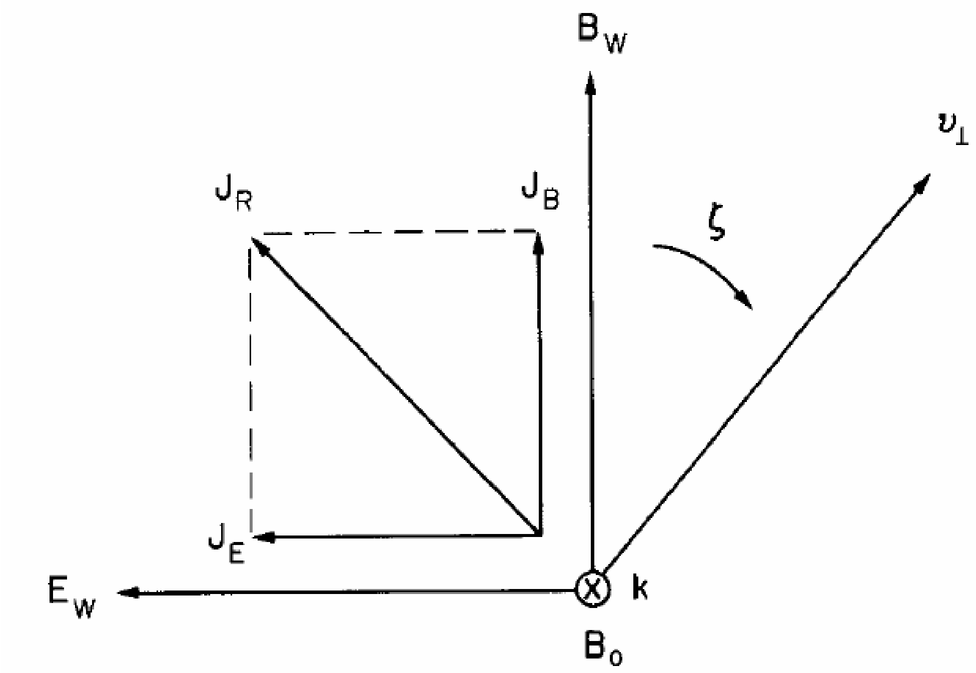
θ
0
π/2
π
3π/2
2π
ζ

# Nonlinear Wave Growth due to Formation of Electromagnetic Electron Hole



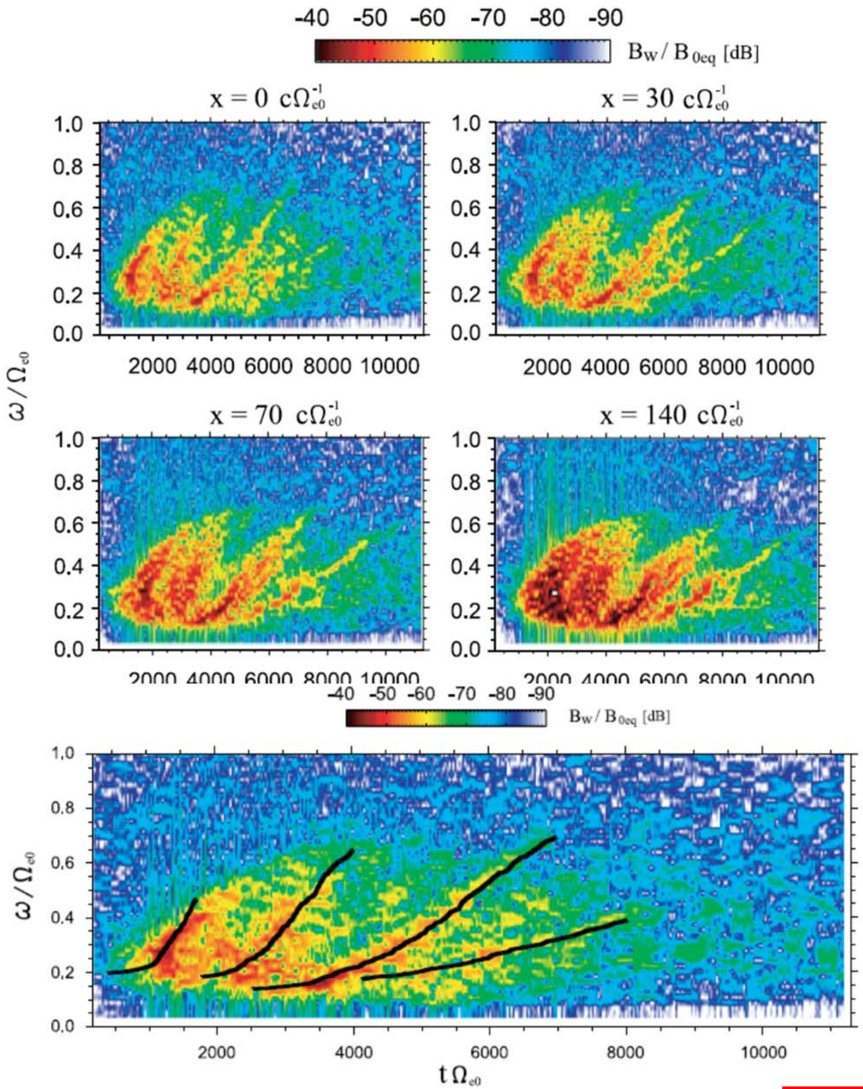




$J_E < 0$ : Wave Growth

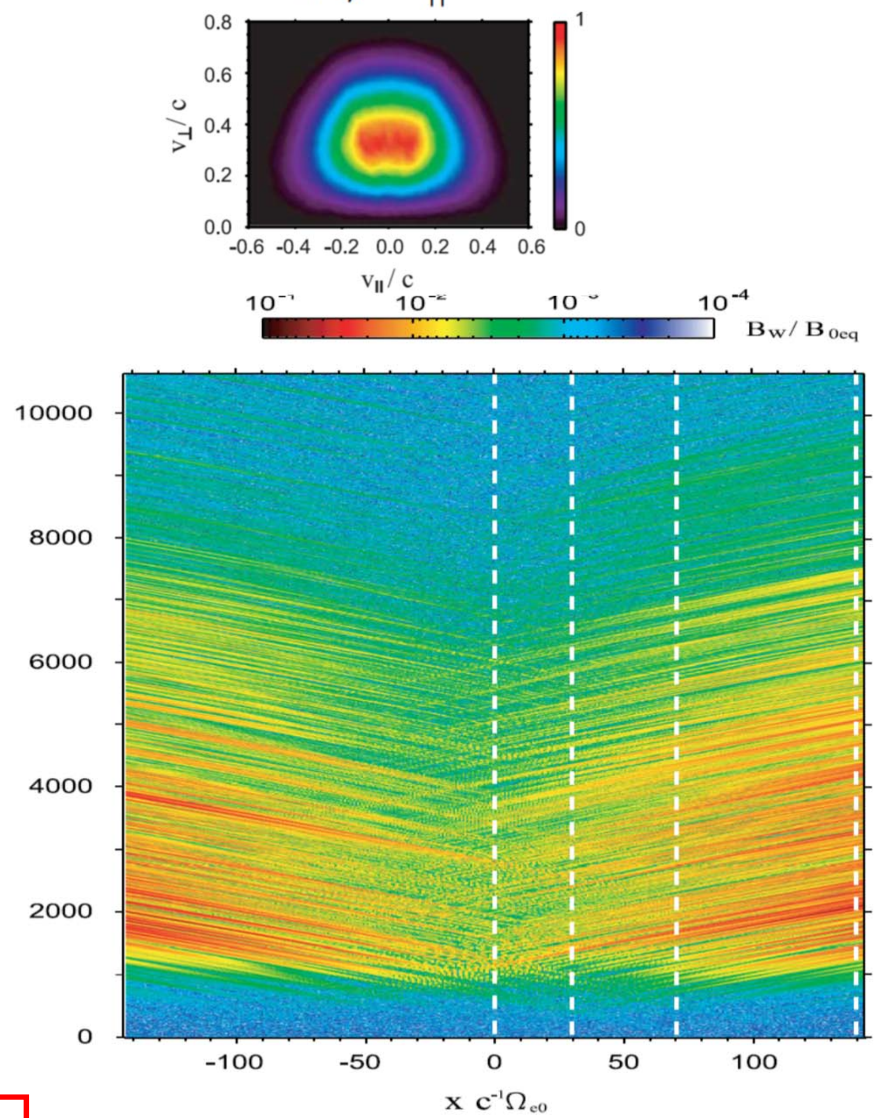
Maximum $-J_E$ $S_{EQ} = -0.4$

[Omura, Katoh, Summers, JGR, 2008]

# Full-Particle Simulation of Chorus Emission



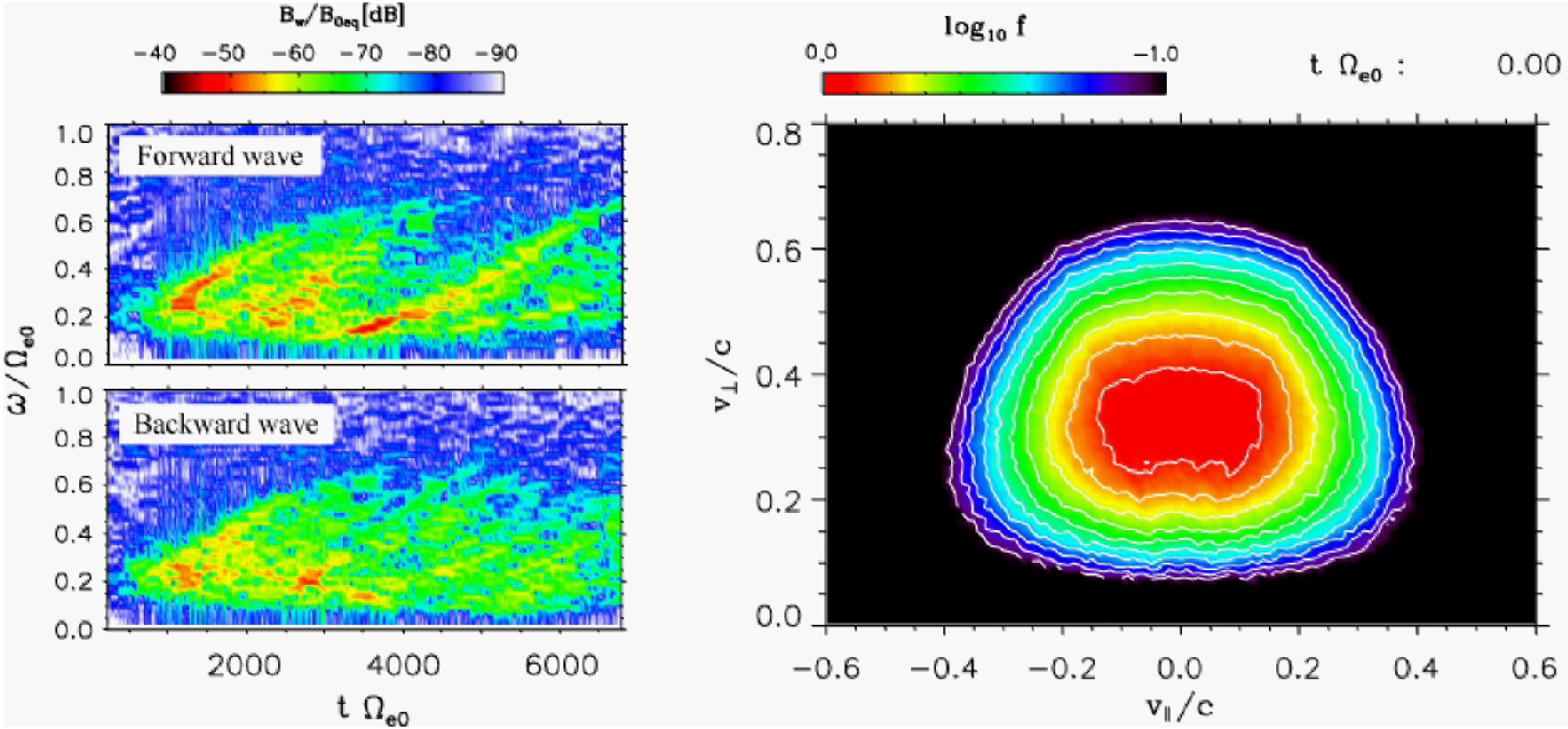

$T_\perp / T_\parallel$ = 3




$$\frac{\partial\omega}{\partial t} = \frac{0.4\,\delta}{\gamma\,\xi}\frac{V_{\perp 0}}{c}\frac{\omega}{\Omega_{e0}}\left(1-\frac{V_R}{V_g}\right)^{-2}\frac{B_w}{B_{0eq}}\Omega_{e0}^2$$

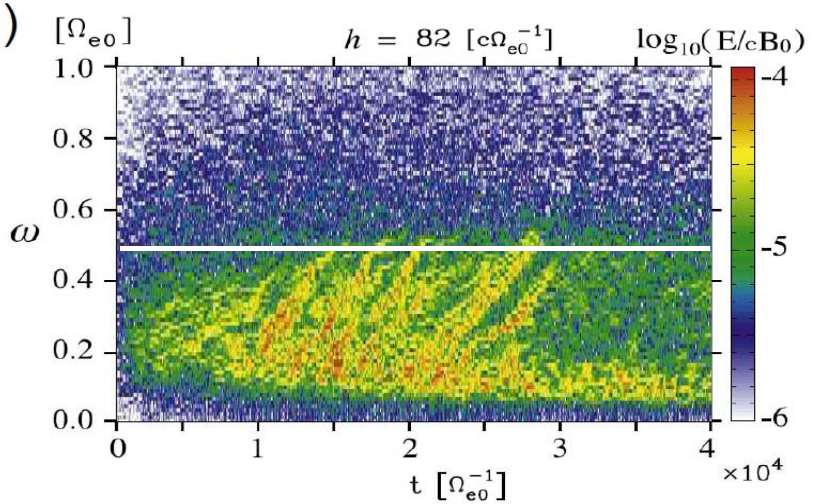
[Hikishima et al., JGR, 2009a]

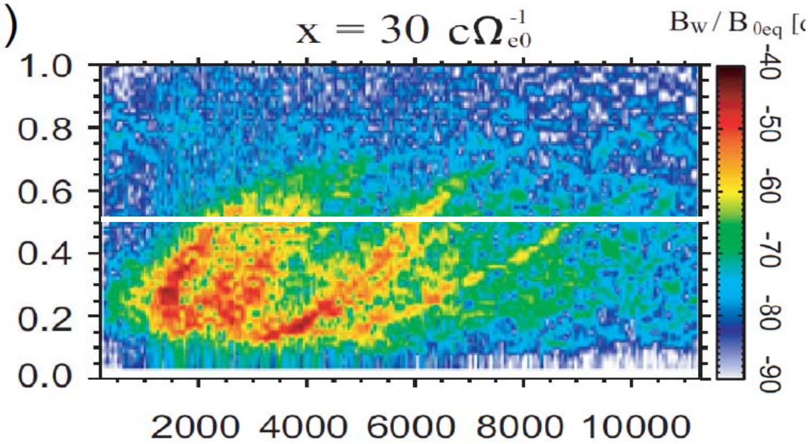
Formation of Pancake Distribution through generation of Chorus emission




[Hikishima et al., JGR, 2009b]

## Simulations :
## parallel propagation

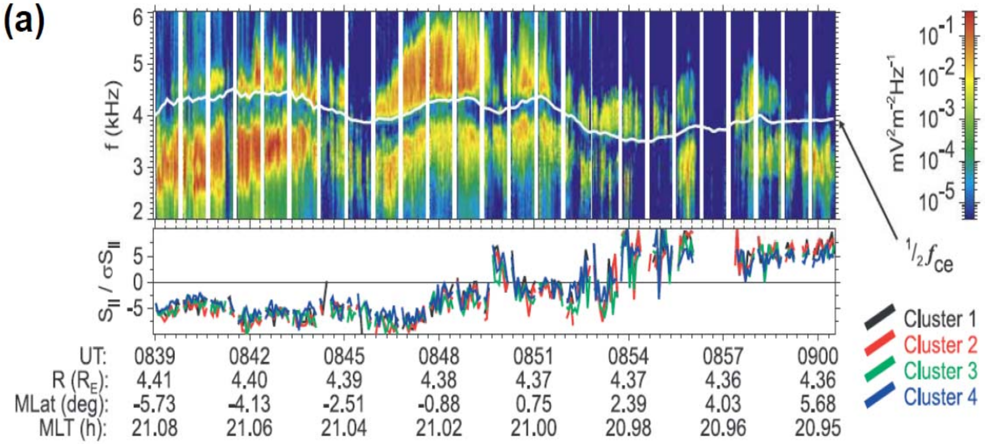
(a)


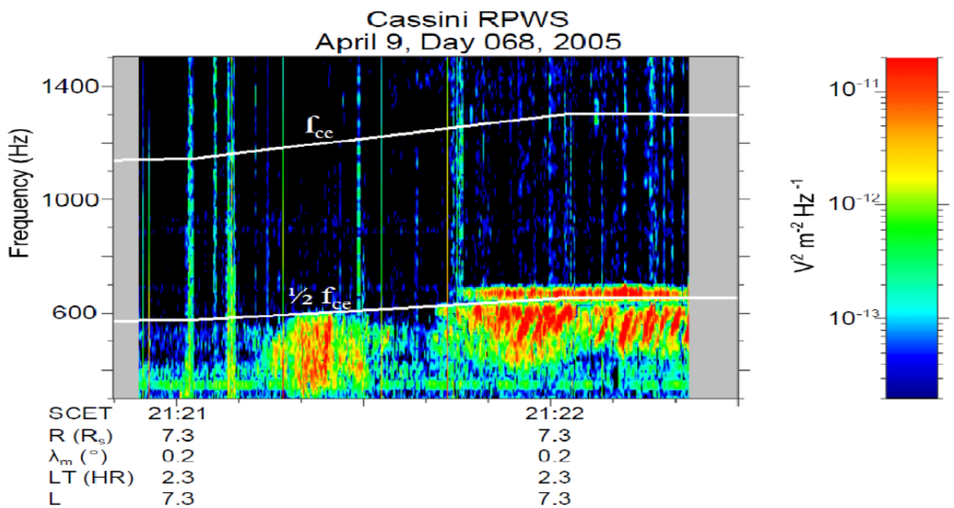

(b)



[*Omura et al., JGR, 2008* ]

[*Hikishima et al., JGR, 2009* ]

## Observations:
## oblique propagation

(a)



(b)



[*Santolik, et al., JGR, 2003* ]

[*Hosphodarsky et al., JGR, 2008* ]

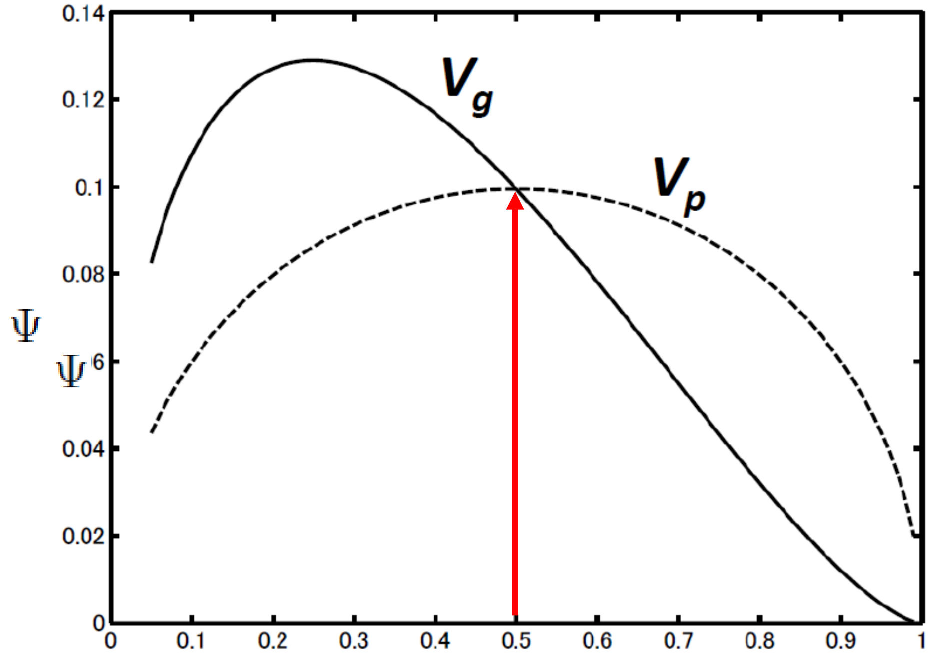
Quasi-parallel Propagation (Oblique) $\sin^2\Psi << 1$

$\Psi$: Wavenormal Angle

$$E_{w\|} = \frac{\omega \sin\Psi}{\delta^2\Omega_e - \omega} E_w$$

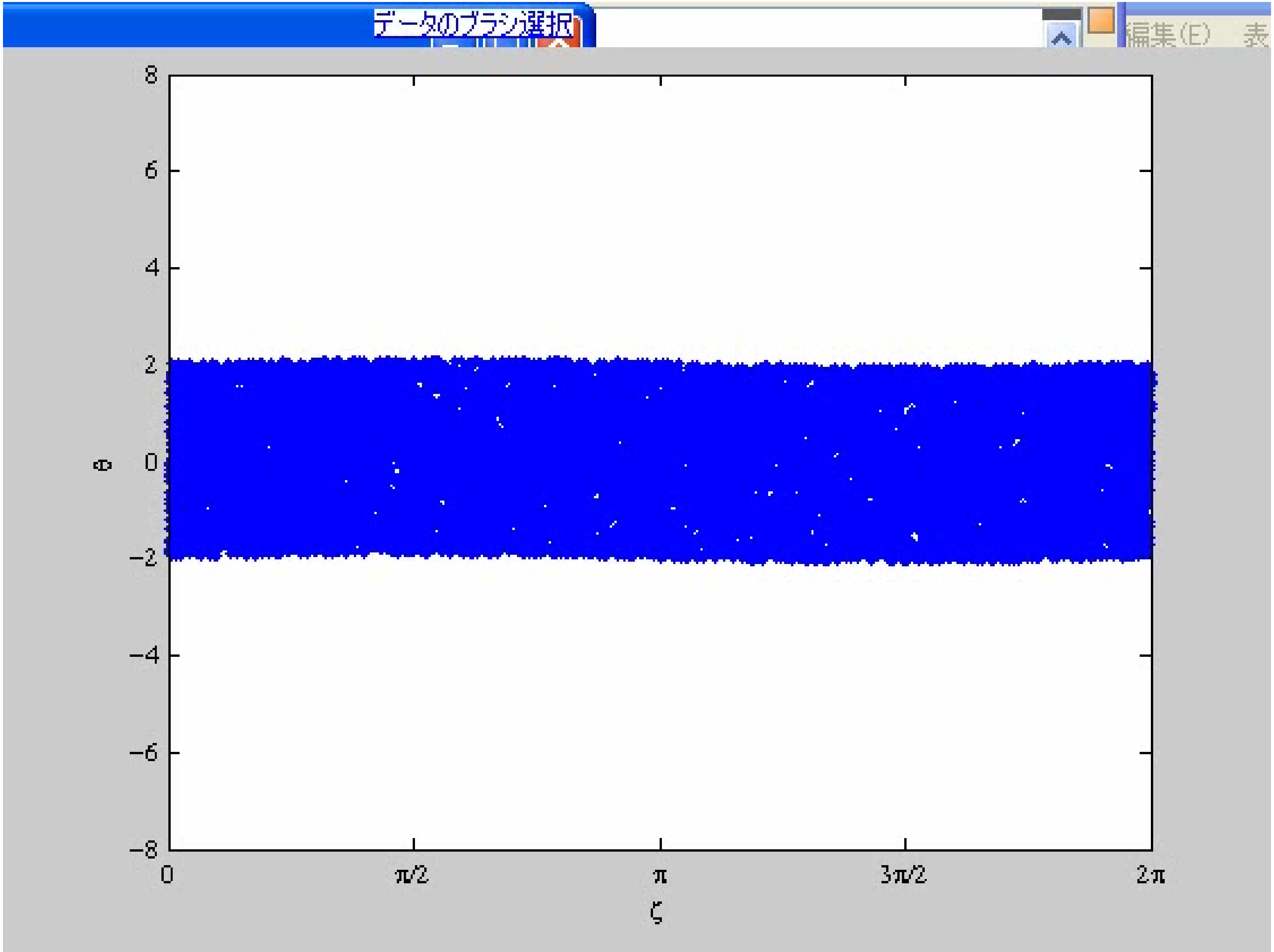
With $\omega = 0.5\Omega_e$

$$V_g = V_p$$




$$\tilde{v}_\| = v_\| - V_p$$

$$\frac{d\tilde{v}_\|}{dt} = -\frac{eE_{w\|}}{\gamma m_0}\sin\phi - \frac{v_\perp^2}{2\Omega_e}\frac{\partial\Omega_e}{\partial h}$$



データのブラシ選択
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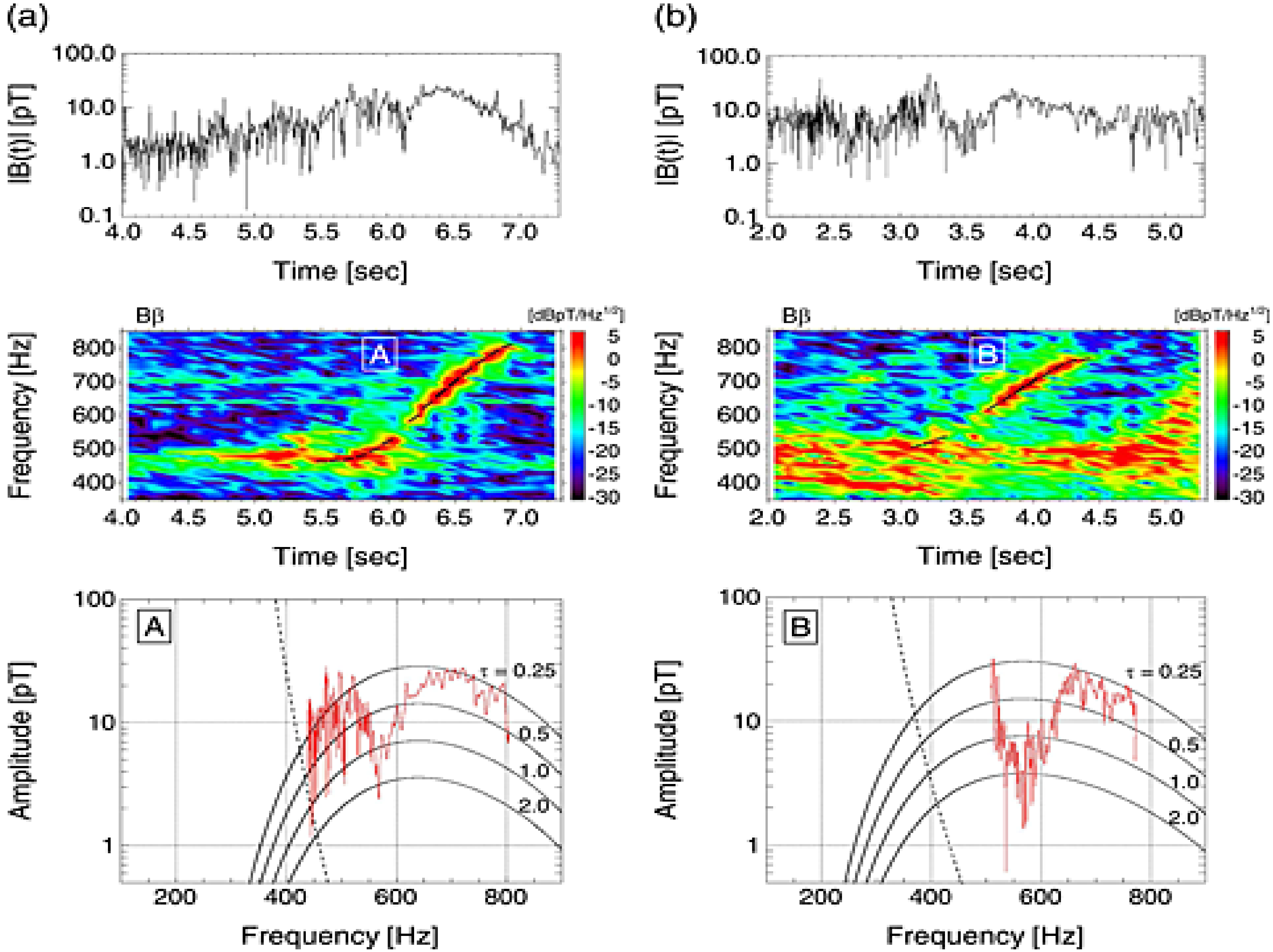
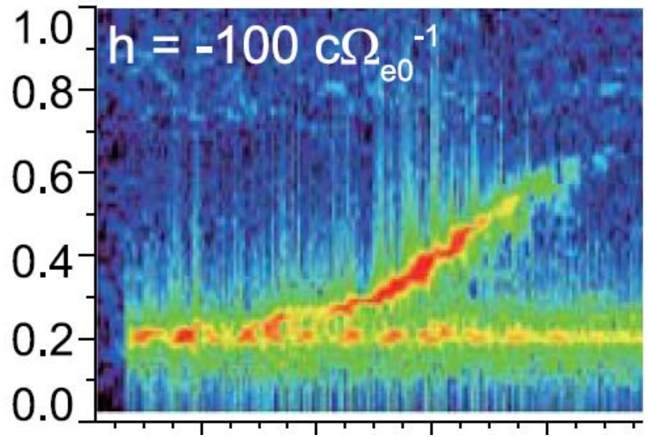
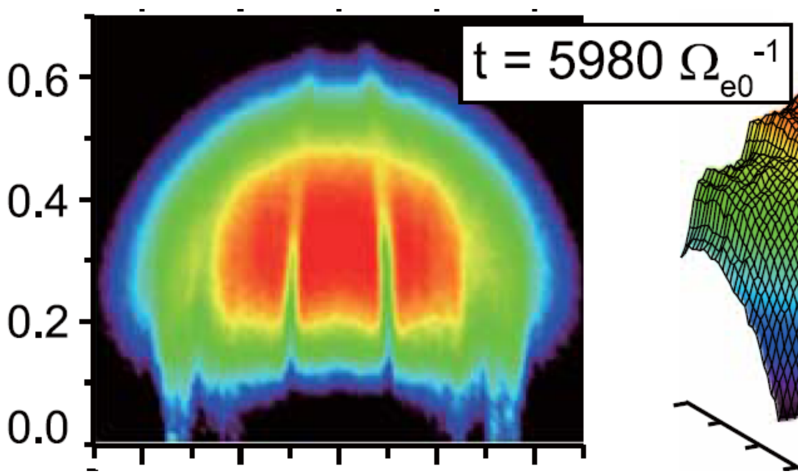
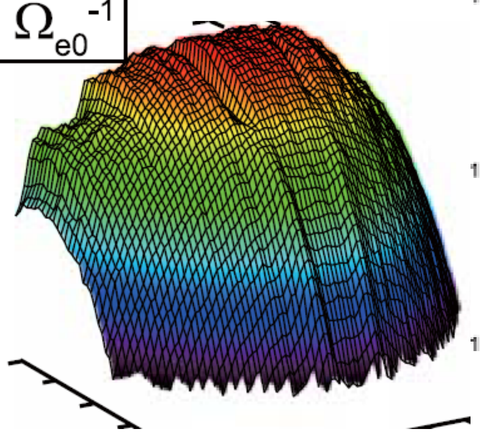


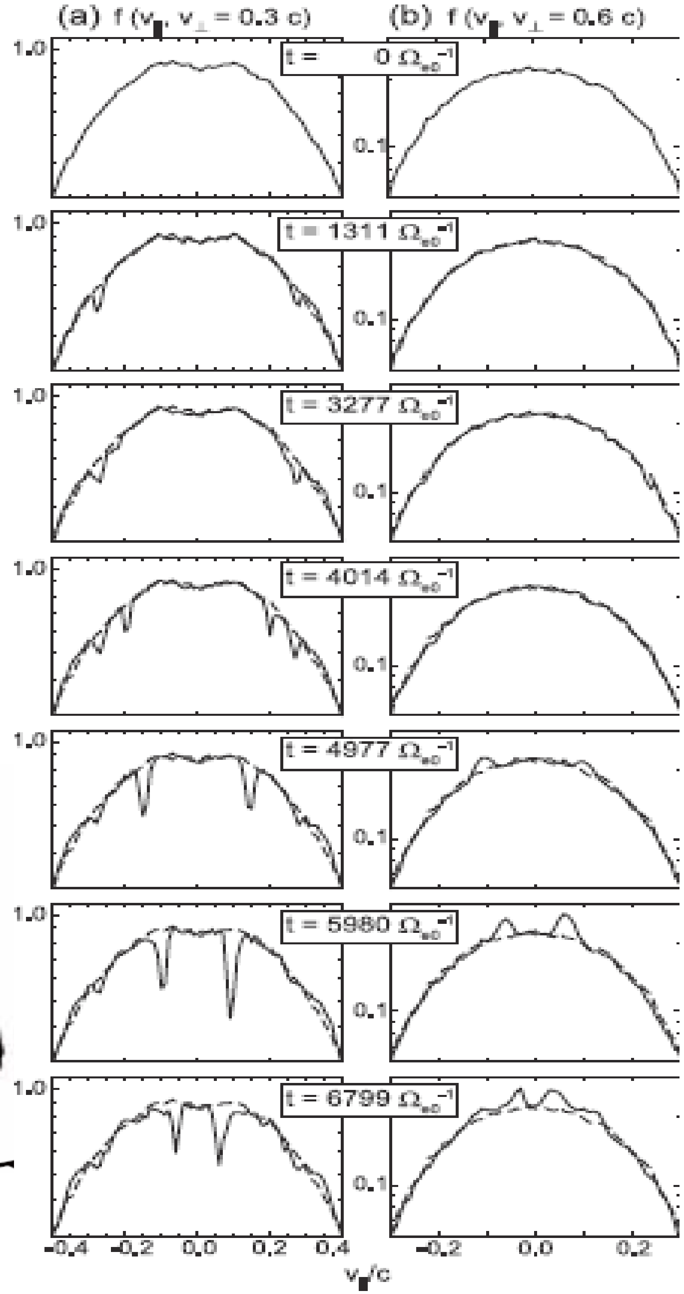

[*Yagitani et al., JGR, 2014*]

# Formation of Electron Hole and Bump



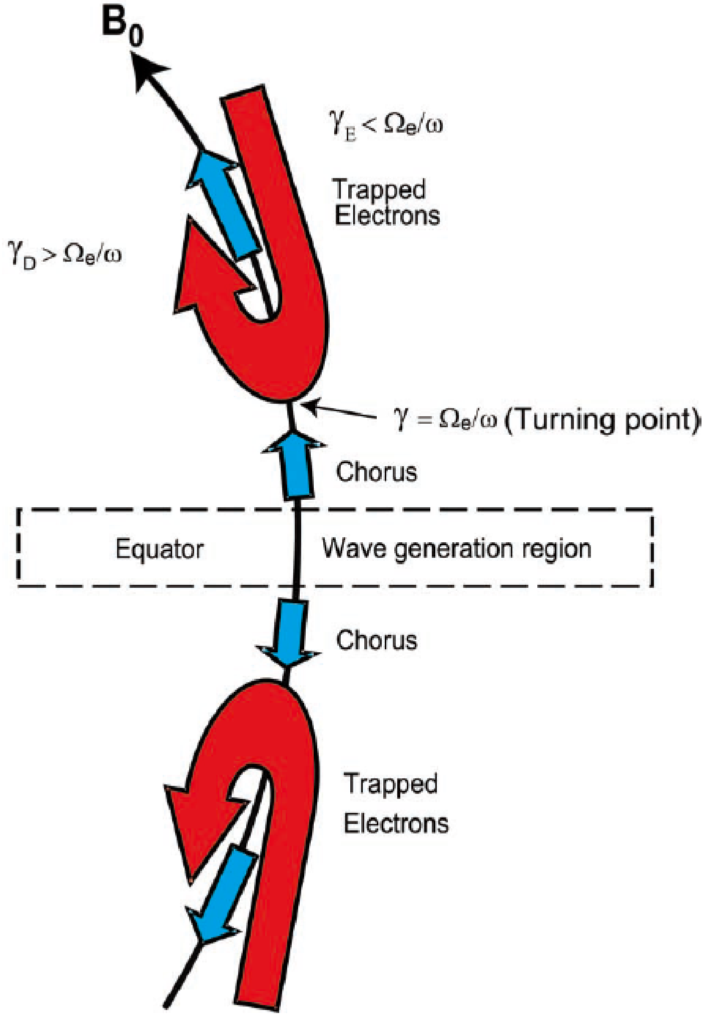



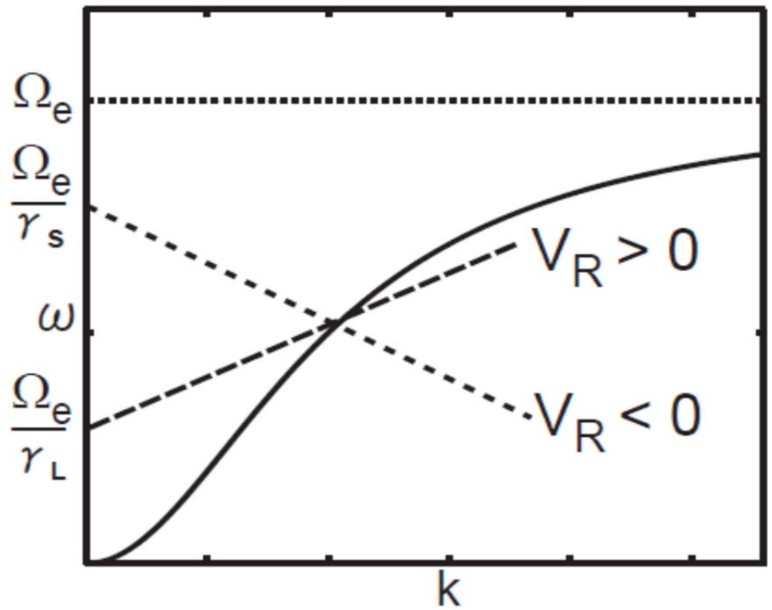



[*Hikishima et al., JGR, 2010* ]

# Relativistic Turning Acceleration




$$V_R = \frac{\omega}{k}\left(1 - \frac{\Omega_e(h)}{\omega\gamma}\right)$$



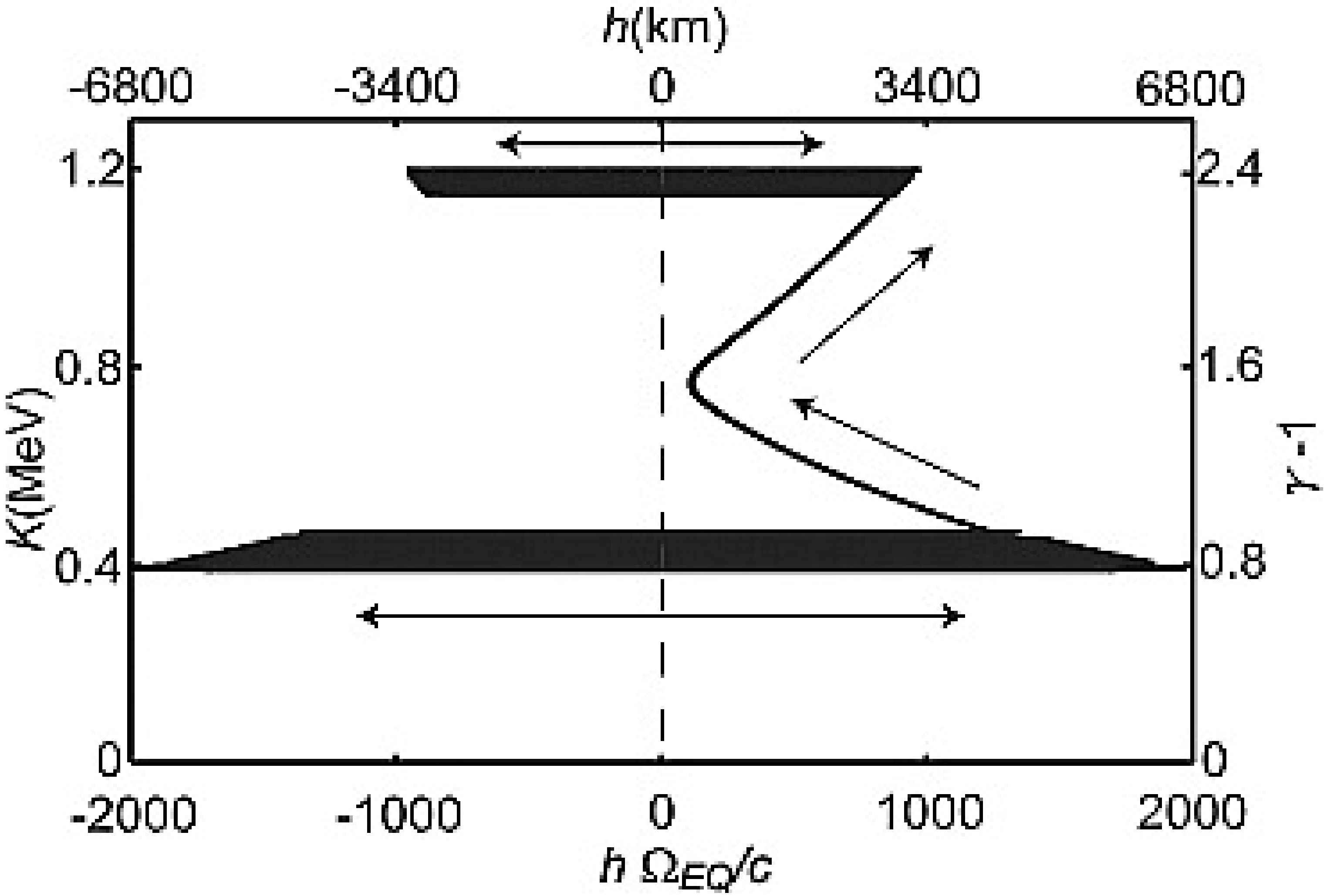

Ultra-Relativistic Acceleration

$\gamma \geqq \Omega_e(h)/\omega \Rightarrow V_R \geqq 0$

[Omura , Furuya, Summers, JGR, 2007]

**Trajectories of Resonant Electrons (400 keV)**

Relativistic Turning Acceleration （RTA）




**Bw = 125 pT**

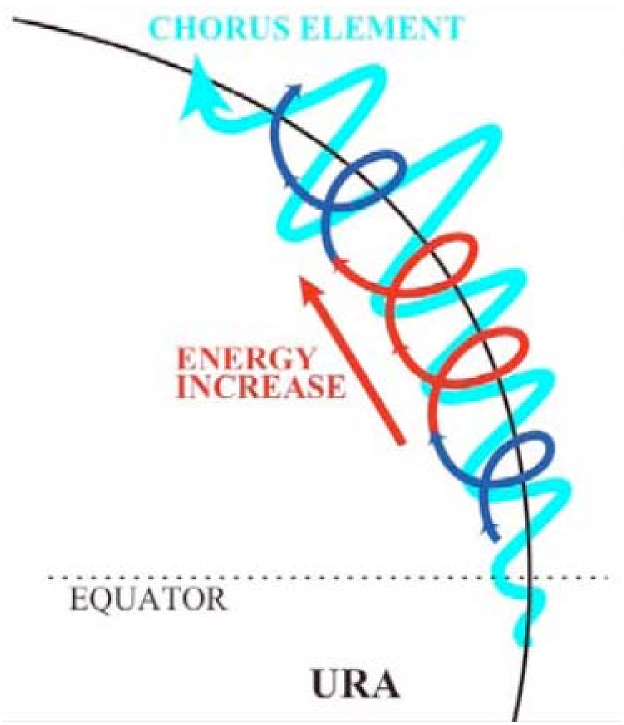
$\omega_p = 2.0\Omega_{e0}$ $\omega = 0.4\Omega_{e0}$

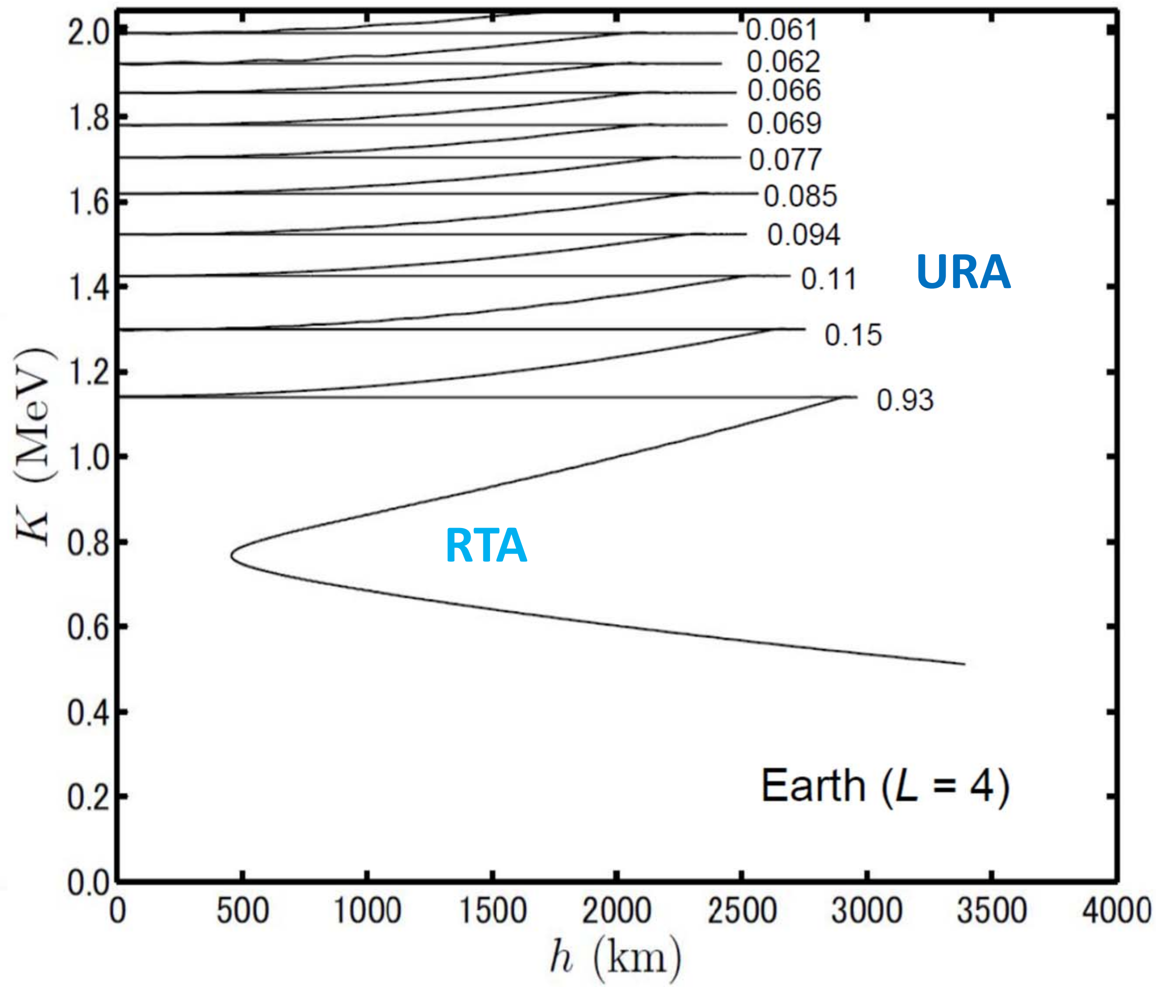
[*Omura, et al., JGR, 2007*]

# Ultra-Relativistic Acceleration (URA)

$$\gamma_0 > \Omega_{EQ}/\omega$$

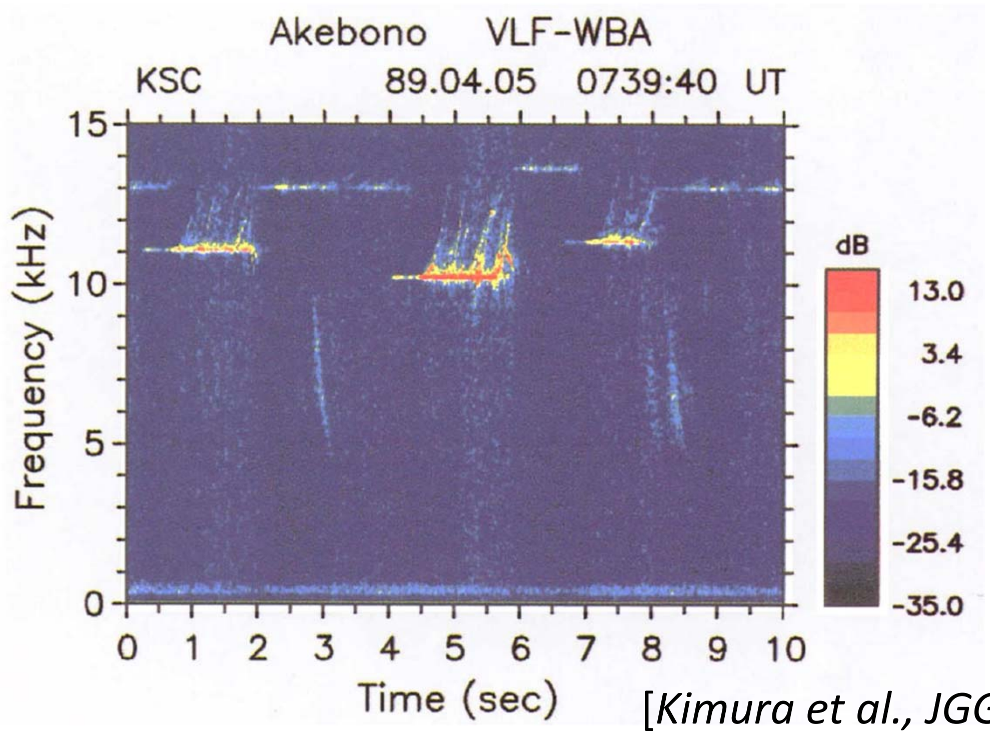
$$V_{R0} > 0$$



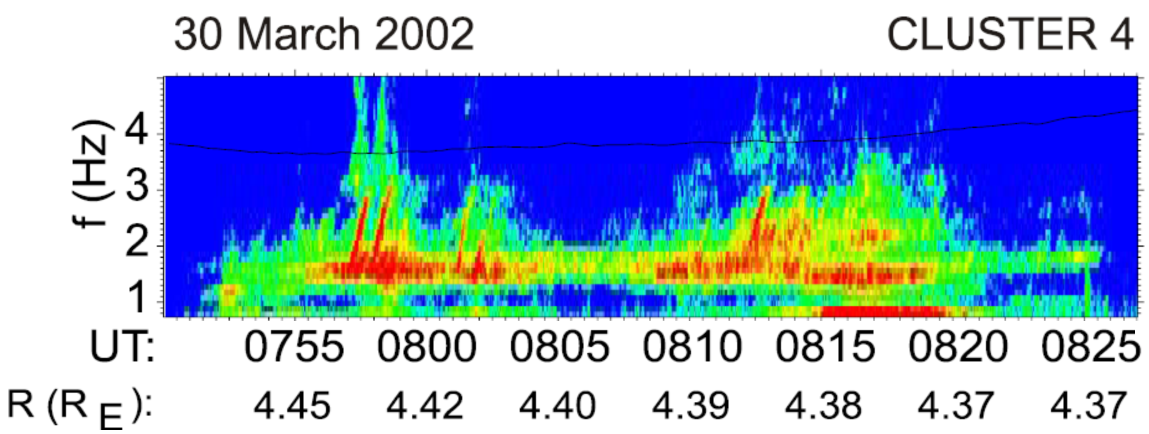

[Summers and Omura, GRL, 2007]

# Whistler-mode Triggered Emissions

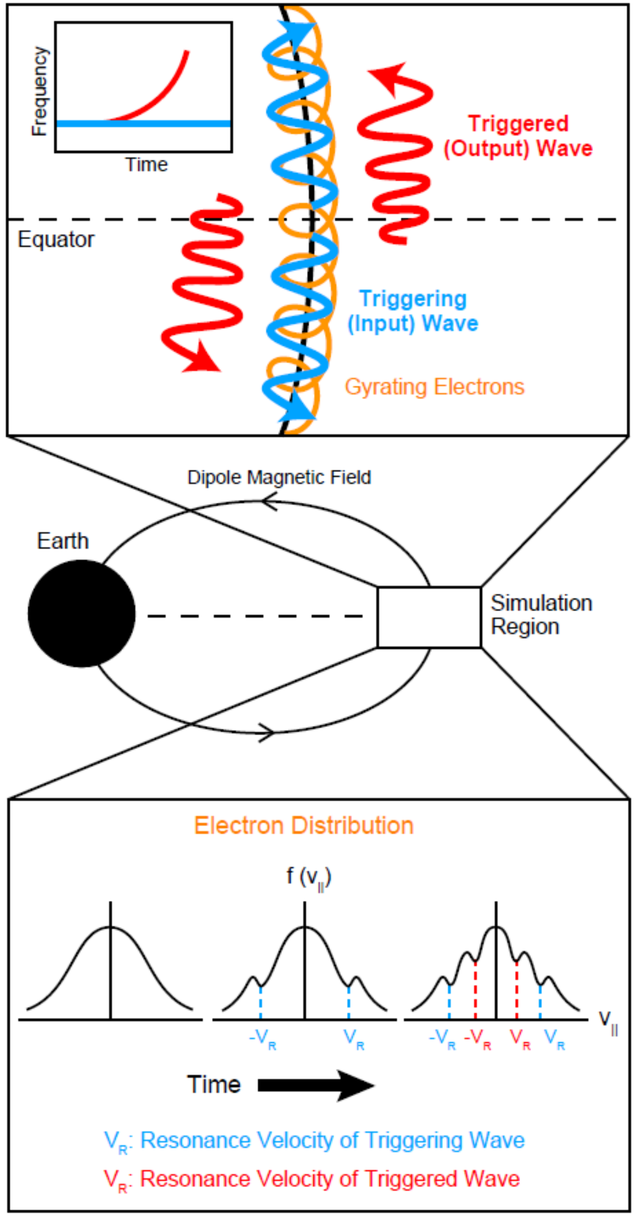



[*Kimura et al., JGG,* 1990]

# EMIC Triggered Emissions

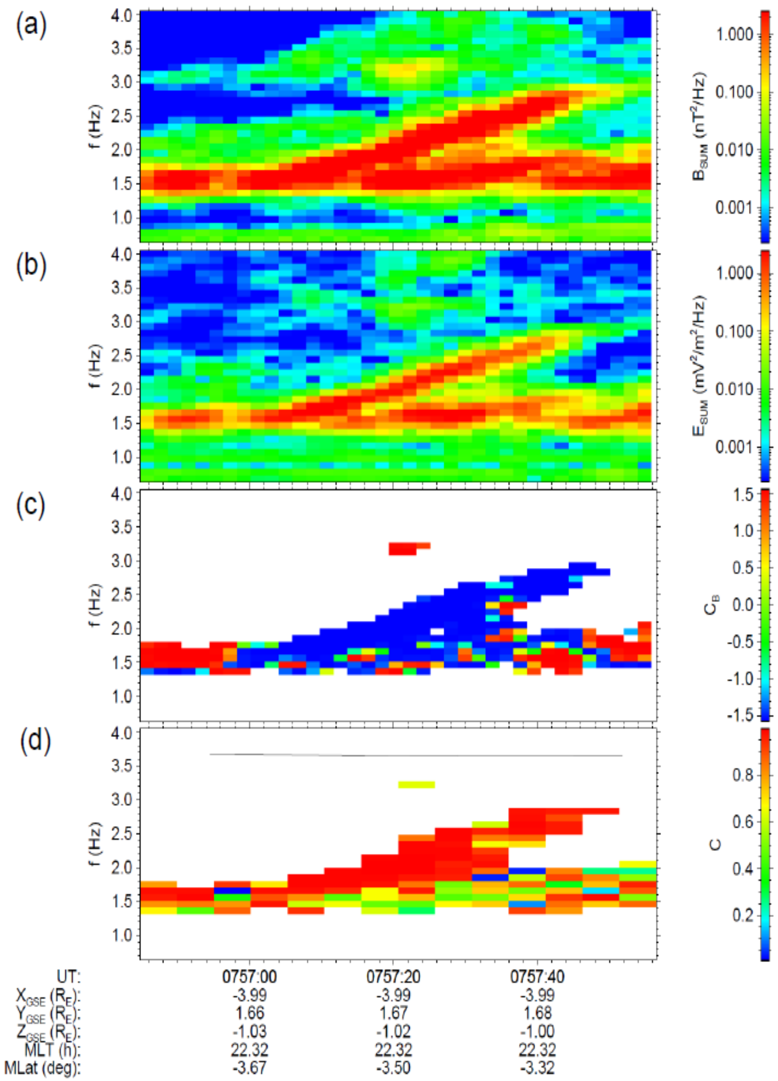



[*Pickett et al., GRL, 2010*]



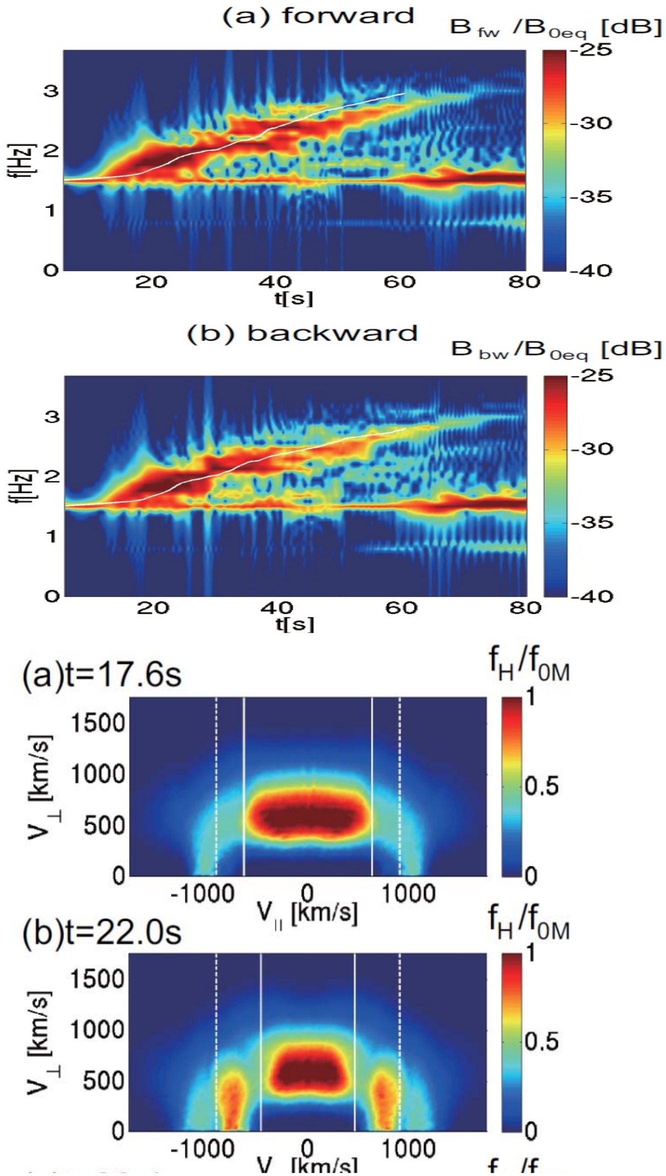
# EMIC Triggered Emissions

**Cluster Spacecraft Observation**

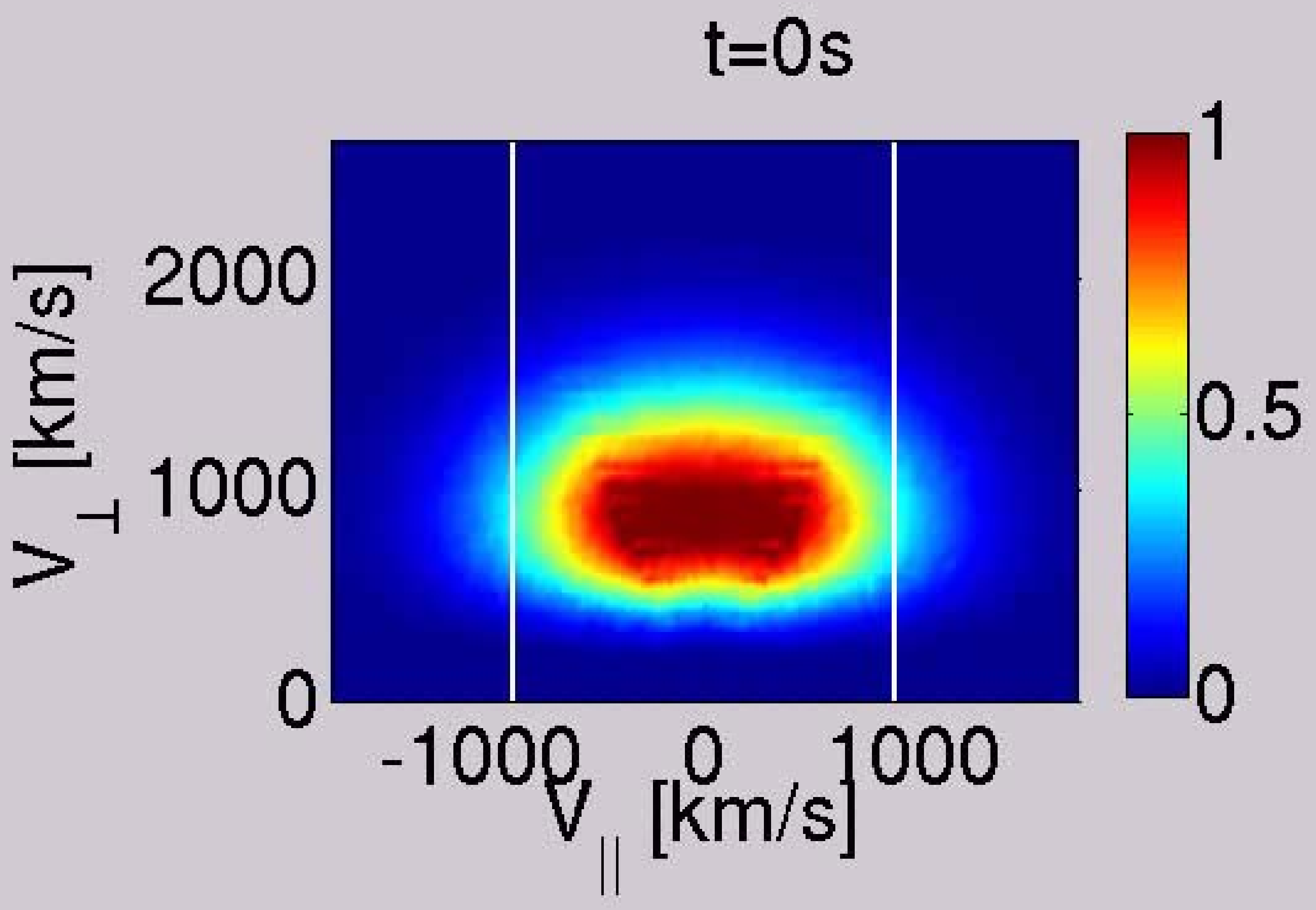



[*Omura et al., JGR, 2010*]

**Hybrid Code Simulation**

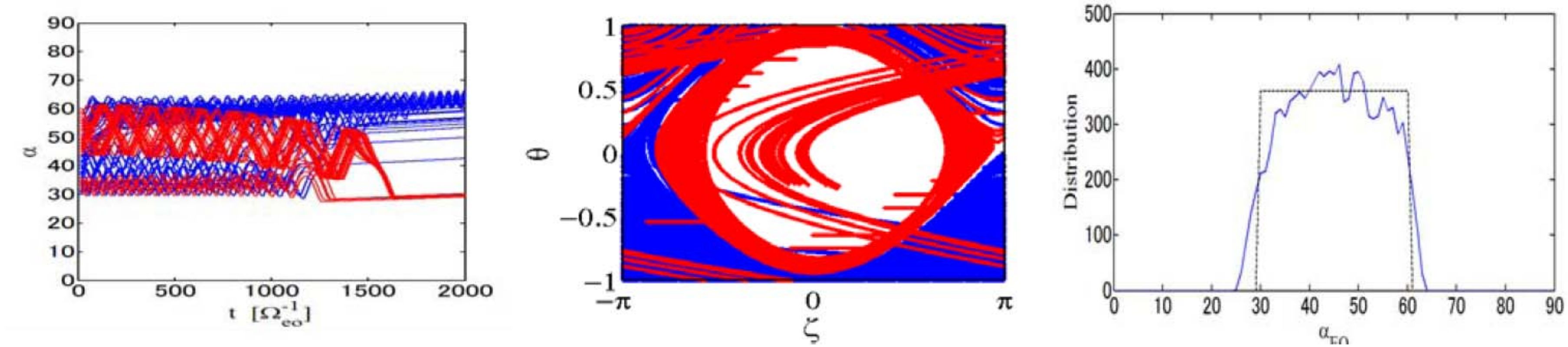



[*Shoji and Omura, JGR, 2011*]

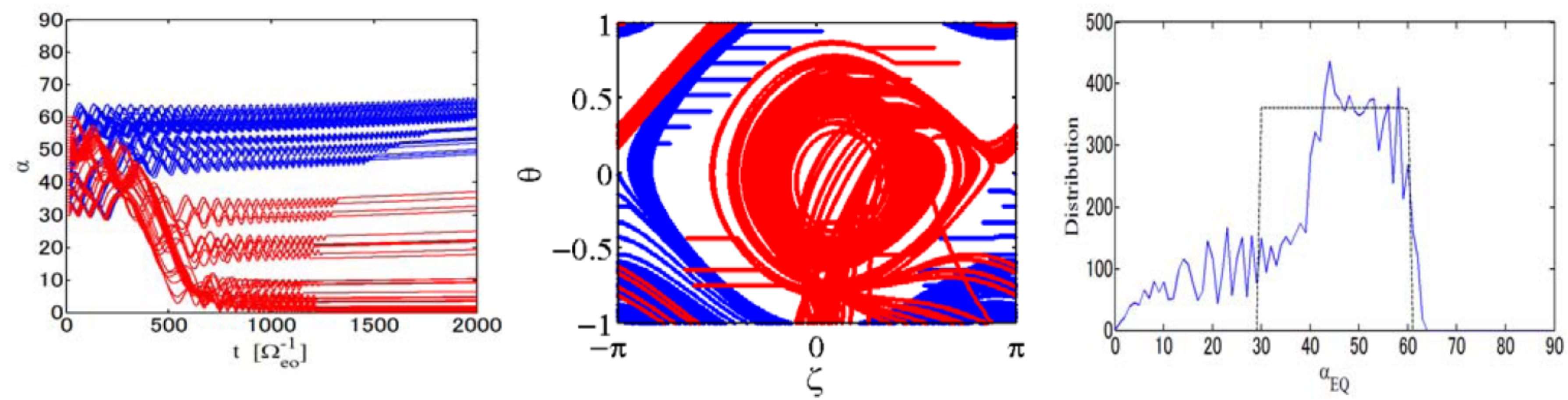

t=0s
2000
1000
0
-1000
0
1000
V_⊥ [km/s]
V_|| [km/s]
1
0.5
0

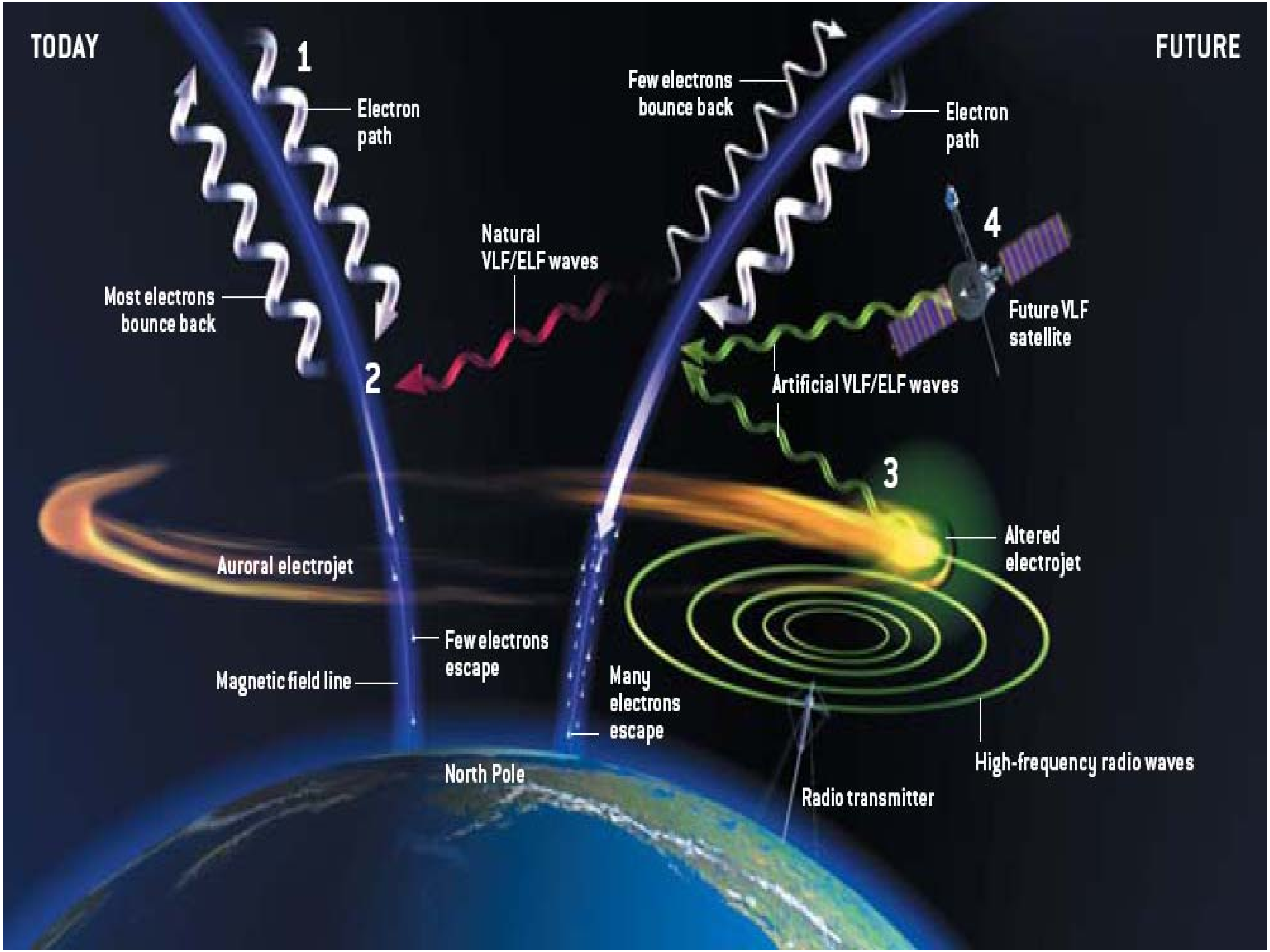
# Test Particle Simulation of EMIC Wave-Electron Interaction

(a) Case A ( 0.98MeV, 2.8Hz-2.8Hz, a = 0.8e-7 )



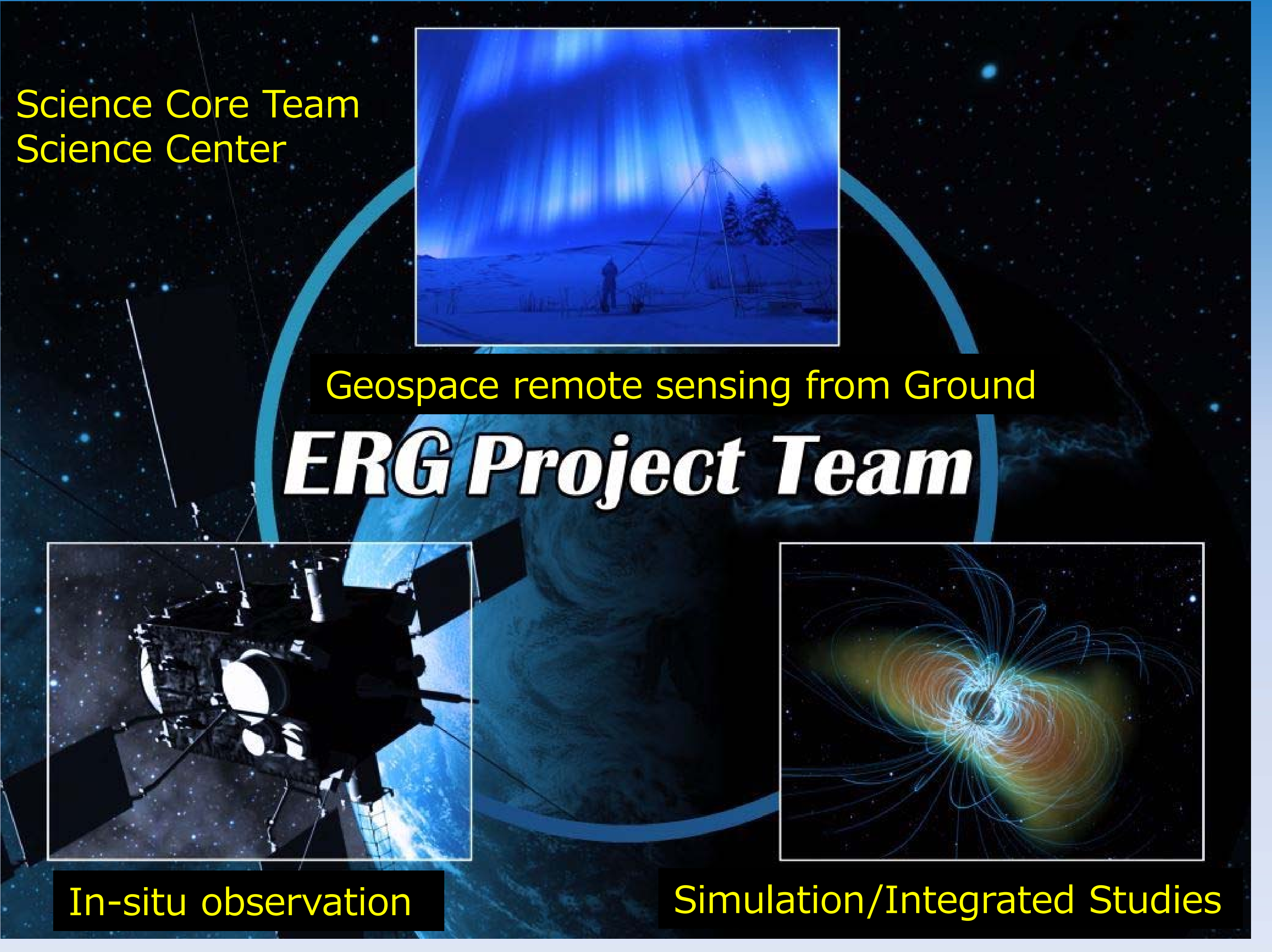

(d) Case D ( 0.98MeV, 2.8Hz-1.7Hz, a = 0.8e-7 )





TODAY
FUTURE
1
Electron path
Most electrons bounce back
2
Natural VLF/ELF waves
Few electrons bounce back
Electron path
4
Future VLF satellite
Artificial VLF/ELF waves
3
Altered electrojet
Auroral electrojet
Few electrons escape
Magnetic field line
Many electrons escape
North Pole
Radio transmitter
High-frequency radio waves

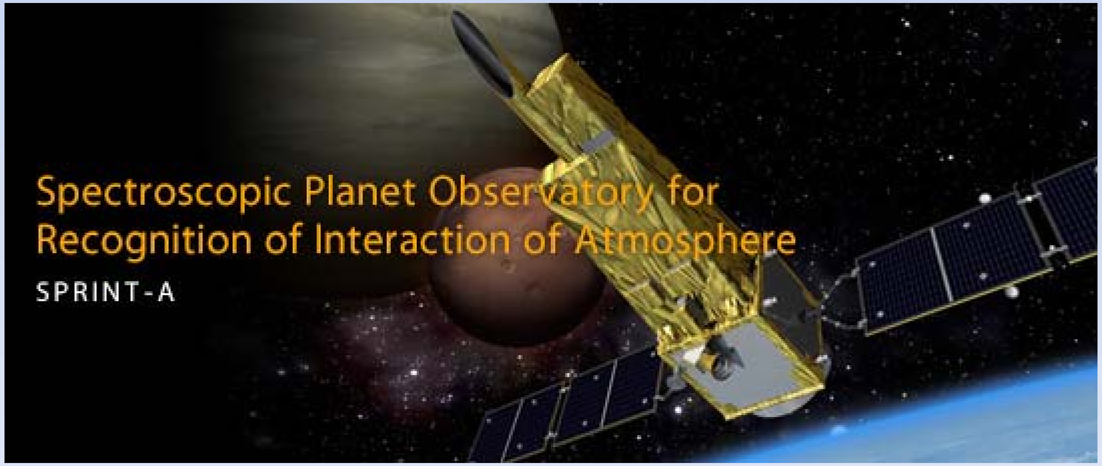
## *2. ERG* Project Group



# First launch of the Epsilon launch vehicle on September 13, 2013.

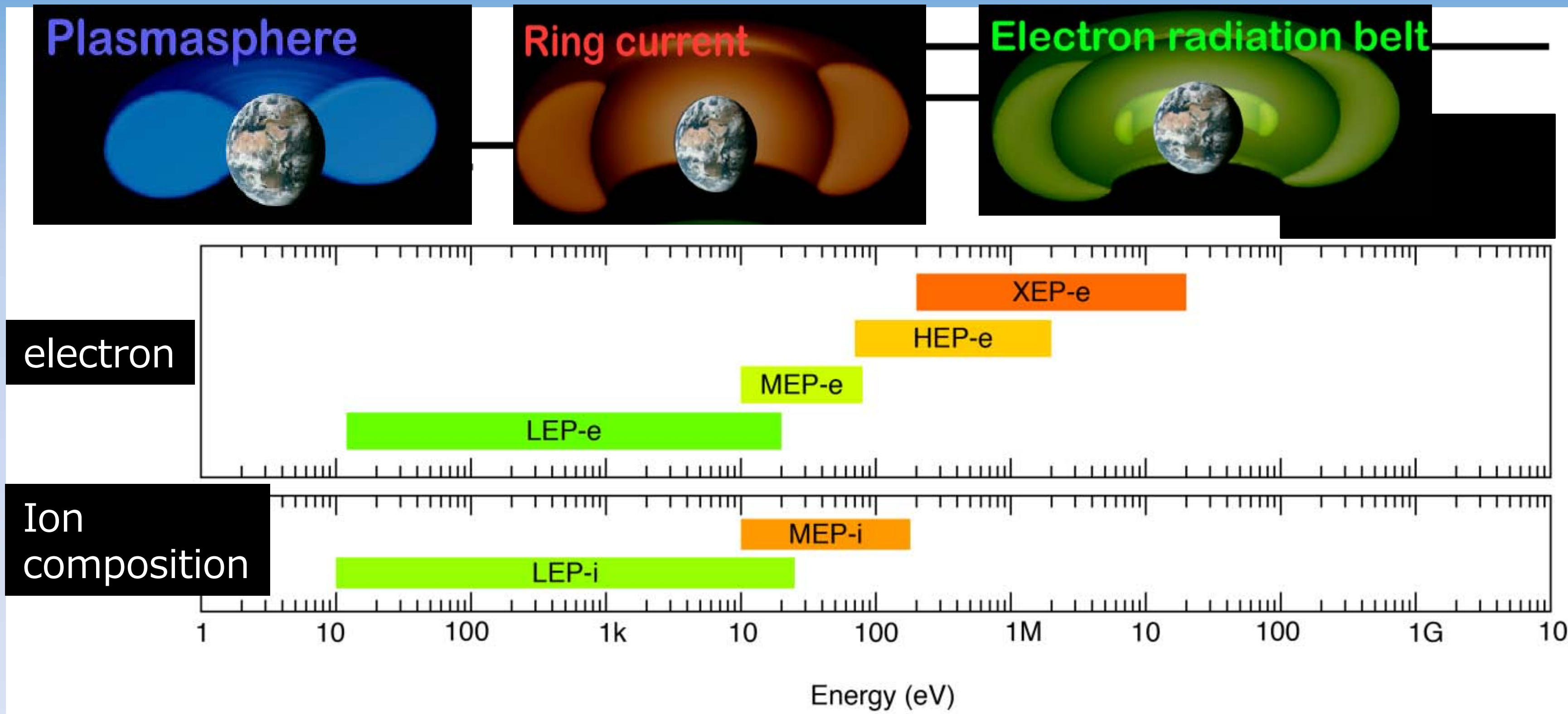

- Hisaki [Spectroscopic Planet Observatory] was successfully launched Epsilon that is newly developed solid rocket on Sep. 13, 2013.

- ERG is the second M-class mission to be launched by Epsilon.



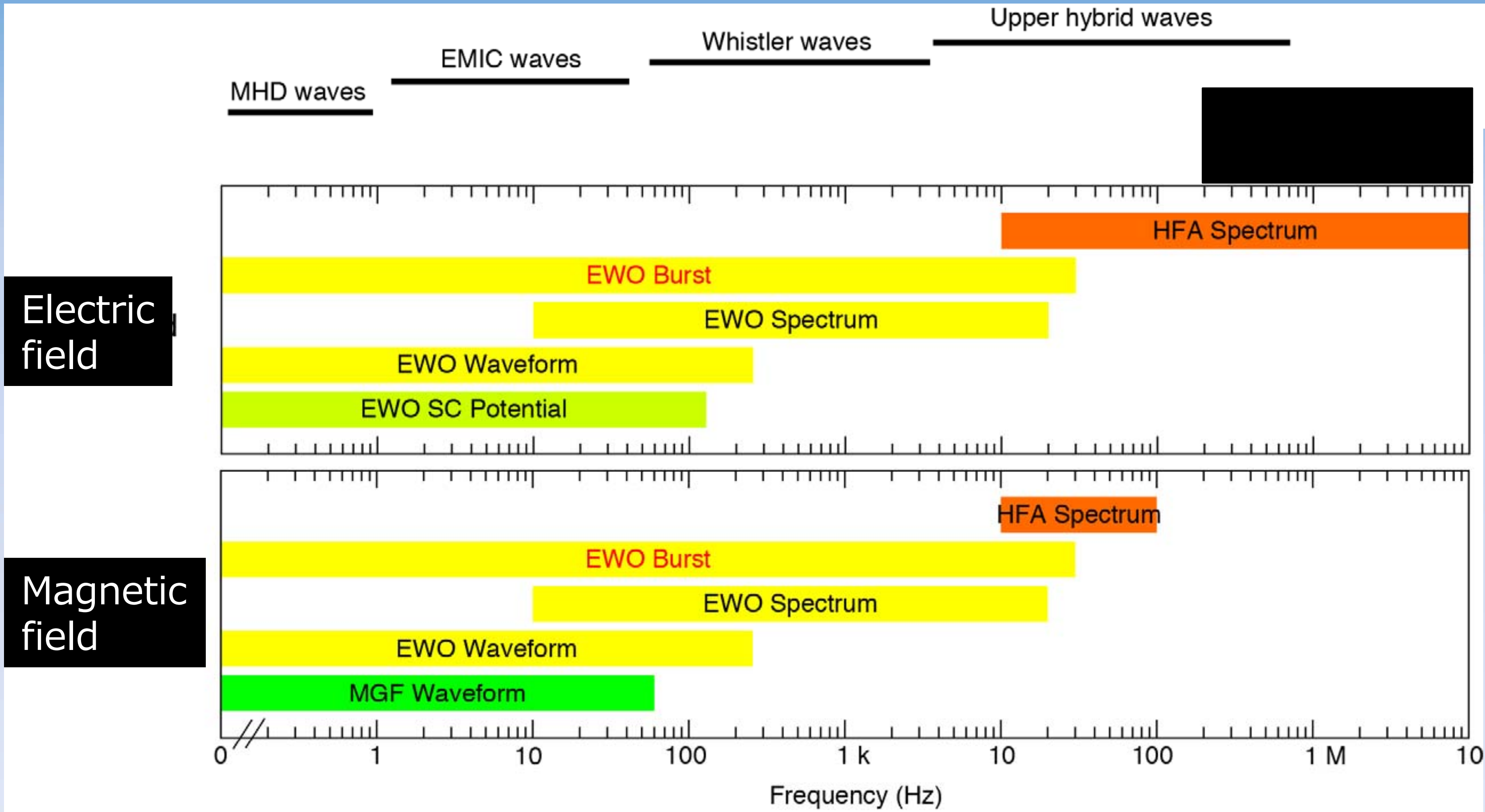
# *ERG* : plasma & particles

## PPE: Plasma and Particle Experiment Suite



# *ERG*: Field and Waves

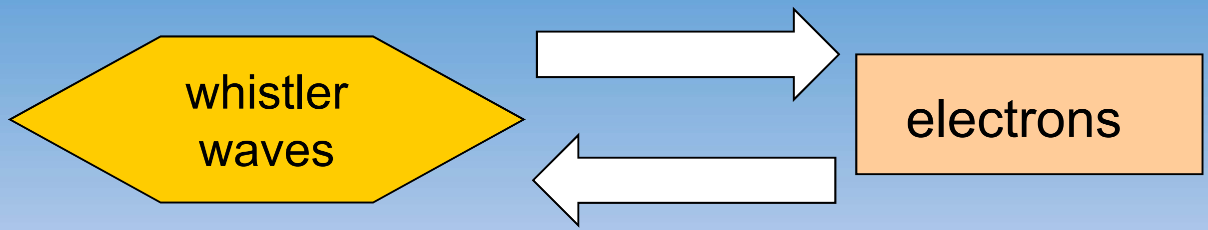
PWE: Plasma Wave and Electric Field Experiment
MGF: Measurement of Geomagnetic Field



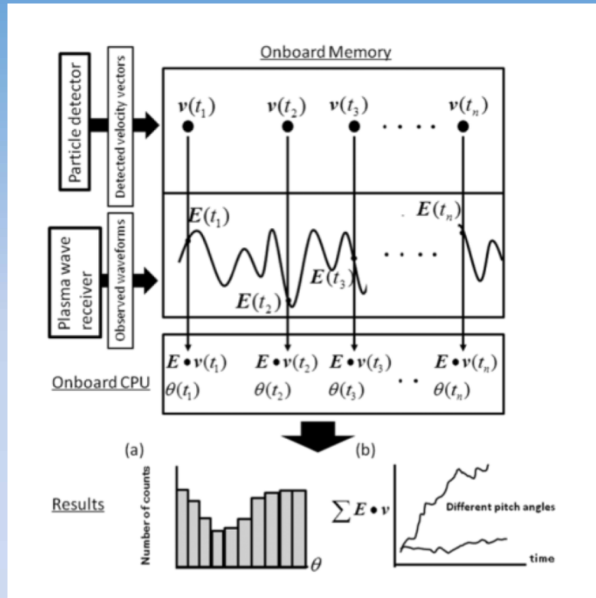
# Wave-Particle Interaction Analyzer

## S-WPIA: Software-type Wave Particle Interaction Analyzer

Direct measurement of energy transfer between whistler waves and electrons is essential to understand wave-particle interactions.




$$\frac{dK}{dt} = qE \cdot v = |E||v| \sin\theta$$




Phase difference θ determines the direction of energy transfer.
(Electrons generate waves or Waves accelerate electrons)

ERG-satellite will directly measure the energy transfer between whistler waves and electrons in space for the first time.

# Mission Status & Schedule

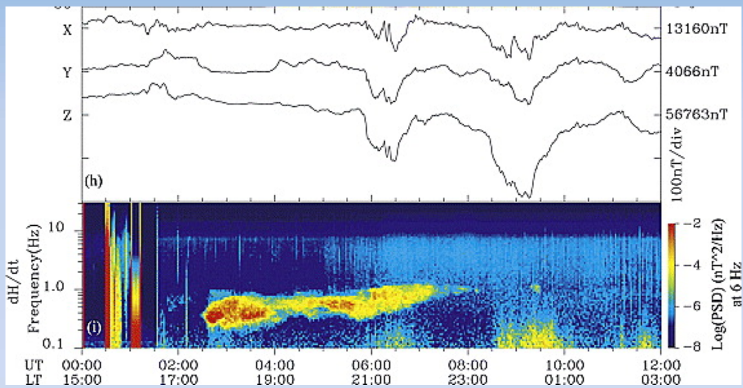
| | | |
|---|---|---|
| FY 2009 | - | Mission Definition Review.<br>System Requirement Review. |
| FY 2011 | - | System Definition Review |
| FY 2012 | - | Preliminary Design Review |
| FY 2013 | - | Critical Design Review |
| FY 2014-15 | - | Development of the flight model |
| 2016 | - | Launch of the satellite |

# The *ERG* ground networks : waves

・Radar Network: SuperDARN network

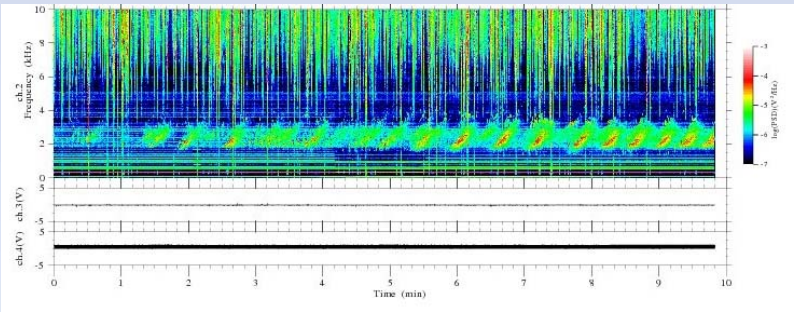
- global convective electric field
- ULF pulsation (Pc5)
- Electric field penetration

・Magnetometer Network : MAGDAS/CPMN,210MM Antarctica Network



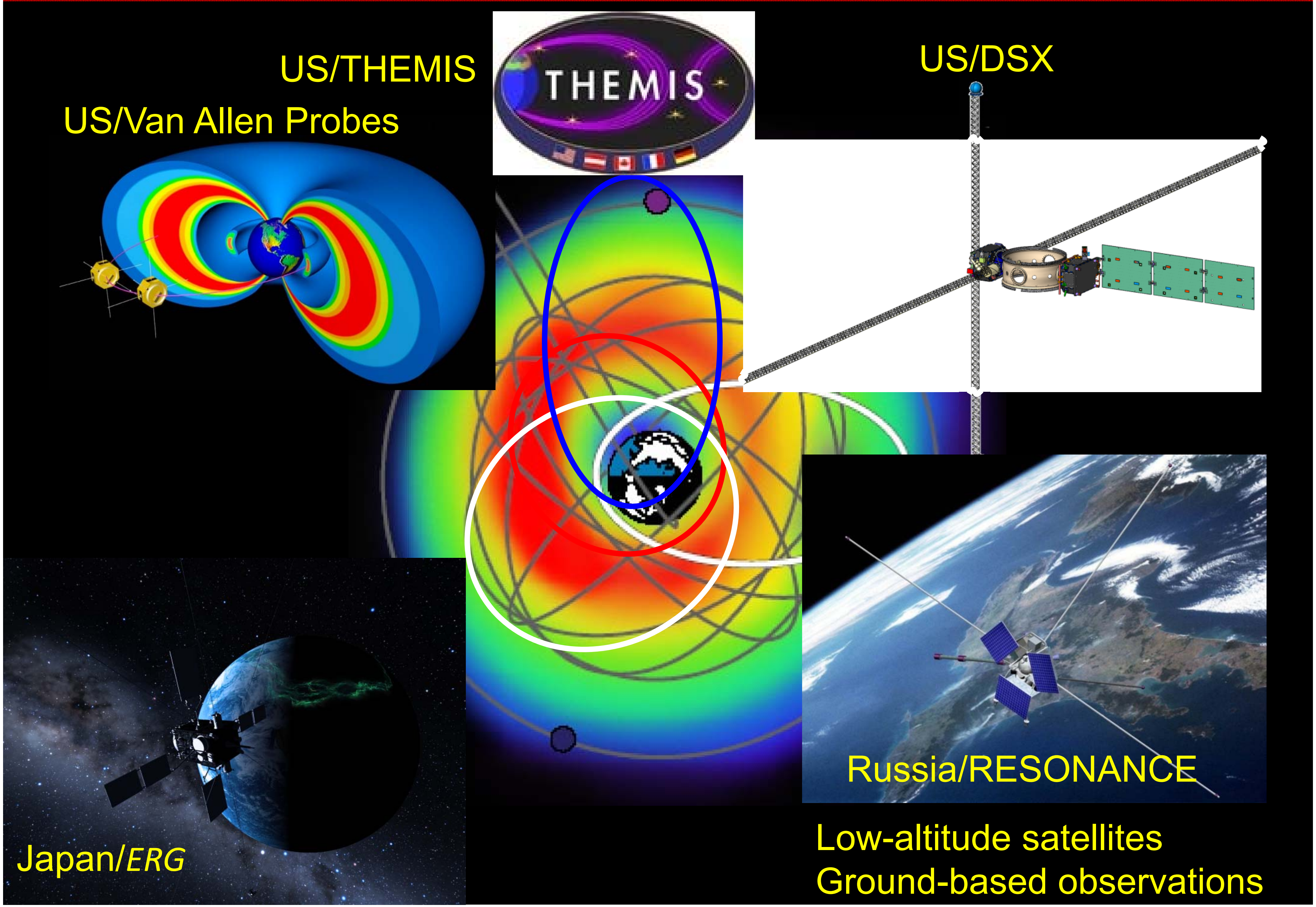

- ionospheric current /ring current.
- ULF pulsation (Pc5).
- EMIC (Pc1).
- diagnostics of plasmasphere

・VLF Network : Canada, Antarctica




- whistler (chorus, hiss).

# 3. International Collaboration: A golden era for geospace



[redacted]

# レポート課題

1．磁気圏のシミュレーションは、宇宙天気における さまざまな擾乱を調べ、予報されることに利用されているが、今後より高い精度で計算可能となった場合、何か別の利用方法がないか考えよ。（地球以外の磁気圏でも可）

2．MHDシミュレーションと粒子シミュレーションの本質的な違いについて簡単に述べよ。